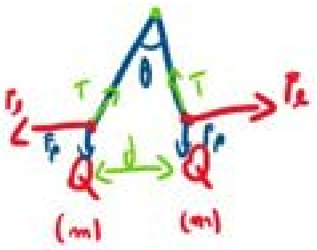
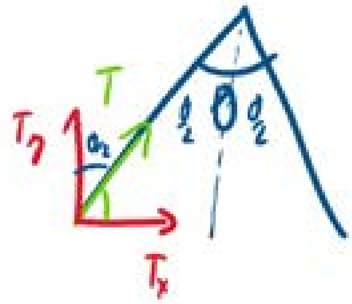
Esercitazione 1

lunedì 27 novembre 2023 18:31





$$\begin{cases} -F_{el,x} + T_x + \cancel{F_{p,x}} = 0 \\ \\ \cancel{F_{el,y}} + T_y - F_{p,y} = 0 \end{cases} \Rightarrow \begin{cases} T \sin(\theta/2) - \dfrac{Q^2}{4\pi\varepsilon_0 d^2} = 0 \\ \\ T\cos(\theta/2) - m \cdot g = 0 \end{cases}$$

$$\Rightarrow \begin{cases} m \cdot g \cdot tg(\theta/2) = \dfrac{Q^2}{4\pi\varepsilon_0 d^2} \\ T = \dfrac{m \cdot g}{\cos(\theta/2)} \end{cases} \Rightarrow \text{"conti"}$$

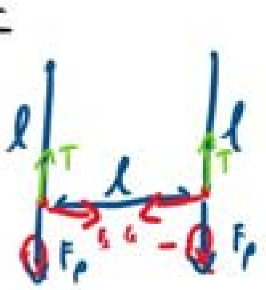
Es 2

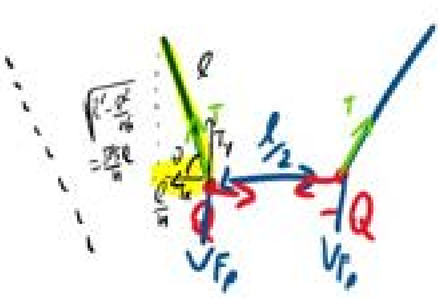
Due sferette di massa $m = 30g$ sono appese a dei fili di lunghezza $l = 40cm$ e distano tra loro l

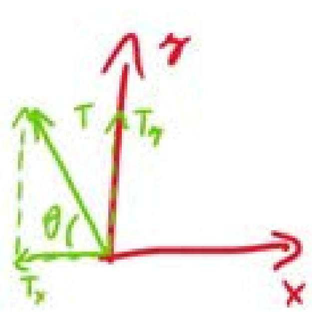
Cariche Q e $-Q$

Trovare $|Q|$ t.c. esiste equilibrio per $d = \frac{l}{2}$

Sol







$\sin(\theta) = \dfrac{\text{cateto opposto}}{\text{ipotenusa}}$

$tg(\theta) = \dfrac{\text{cateto opposto}}{\text{cateto adiacente}}$

$$\Big(T_x + F_{el,x} + \cancel{F_{p,x}} = 0 \qquad \Big(T \cdot \cos(\theta) + \frac{-Q^2}{4\pi\varepsilon_0 (\frac{l}{2})^2} = 0$$

$$\begin{cases} T_x + F_{e,x} + \cancel{F_{p,x}} = 0 \\ T_y + \cancel{F_{e,y}} + F_{p,y} = 0 \end{cases} \Rightarrow \begin{cases} T \cdot \cos(\theta) + \dfrac{-Q^2}{4\pi\varepsilon_0 (\frac{l}{2})^2} = 0 \\ T \cdot \sin(\theta) - m \cdot g = 0 \end{cases}$$

$$\Rightarrow \begin{cases} T \cdot \frac{1}{4} - \dfrac{Q^2}{4\pi\varepsilon_0 (\frac{l}{2})^2} = 0 \\ T \cdot \frac{\sqrt{15}}{4} - 294{,}3 = 0 \end{cases} \Rightarrow \begin{cases} |Q| = \sqrt{\pi \varepsilon_0 l^2 \cdot \frac{294{,}3}{\sqrt{15}}} = \text{"Numero"} \\ T = \dfrac{294{,}3 \cdot 4}{\sqrt{15}} \end{cases}$$

Es 3

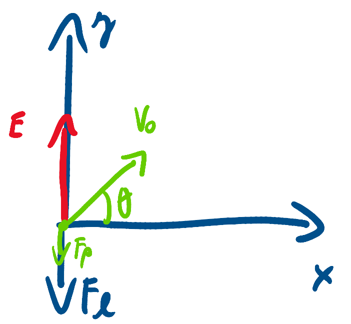
$|V_0| = 5{,}83 \cdot 10^6$

$\theta = 39°$ con l'orizzontale

$E = 7870 \frac{N}{C}$ $(\uparrow)$

Descrivere il moto dell'elettrone

Sol



$F_{TOT} = F_p + F_e = m \cdot g + E \cdot q_{elettrone}$

É Parabolico

Da trovare l'equazione del moto

$$\begin{cases} S_{f,x} = S_0 + V_{0,x} \cdot t \\ S_{f,y} = \cancel{S_0} + V_{0,y} \cdot t + \frac{1}{2} a t^2 \end{cases}$$

$\Rightarrow a = \dfrac{F_{TOT}}{m_{elettrone}}$

$$\int_{t_0}^{t} a \, dt = a t - a t_0$$

$$V = V_0 + a t$$

$$\Rightarrow t = -\frac{V_0}{a}$$

$\blacksquare$

Esercitazione 2

mercoledì 29 novembre 2023 16:37

$\rho = \frac{\partial Q}{\partial V}$

R "raggio"

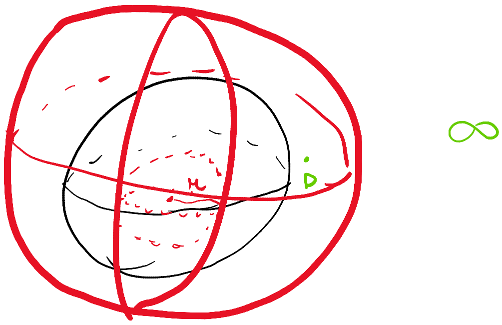
Q

1

$$E = \begin{cases} r < R \\ r = R \\ r > R \end{cases}$$

1)

$$E = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2}$$



$$\boxed{S = 4\pi R^2}$$

$$\phi_S(E) = \frac{Q}{\varepsilon_0} = \int_S \vec{E} \cdot \hat{n} \, ds$$

$\Rightarrow$

$$\frac{Q}{\varepsilon_0} = \int_S \vec{E} \cdot \hat{n} \, ds = E \, 4\pi r^2$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho \cdot V}{4\pi\varepsilon_0 r^2} = \frac{\frac{4}{3}\pi R^3 \rho}{4\pi\varepsilon_0 r^2} = \frac{R^3 \rho}{3\varepsilon_0 r^2} \quad (r > R)$$

Se $r < R$

$$\frac{Q}{\varepsilon_0} = \int_S \vec{E} \cdot \hat{n} \, ds$$

$\Rightarrow$

$$\frac{Q}{\varepsilon_0} = E \, 4\pi r^2$$

$$\Rightarrow \quad E = \frac{Q}{4\pi \varepsilon_0 r^2} = \frac{\rho \cdot V}{4\pi\varepsilon_0 r^2} = \frac{\rho \cdot \frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2} = \frac{\rho \cdot r}{3\,\varepsilon_0}$$

Se $r = R \Rightarrow r < R$

Trovare potenziale per $r < R$, $r = R$, $r > R$

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\frac{L}{q_0} = \int \overset{=E}{\boxed{\frac{F}{q_0}}}\, d\ell = \frac{U_i - U_f}{q_0} = V_i - V_f$$

$$V_\rho - \cancel{V_\infty}^{=0} = \int_\rho^\infty E \cdot d\ell$$

$$V_\rho = \int_\rho^\infty E \cdot d\ell = \int_\rho^\infty \frac{R^3 \cdot \rho}{3\varepsilon_0 r^2}\, dr = \frac{R^3 \cdot \rho}{3\,\varepsilon_0} \int_\rho^\infty \frac{1}{r^2}\, dr$$

$$= \frac{R^3 \cdot \rho}{3\,\varepsilon_0} \left(-\frac{1}{r}\right)\Big|_{\rho = r}^{\infty} = \frac{R^3 \cdot \rho}{3\,\varepsilon_0 r} \qquad (r > R)$$

$$V(R) = \frac{R^3 \rho}{3\varepsilon_0 R} = \frac{\rho R^2}{3\varepsilon_0}$$

Se $r < R$

$$V(r) - V(R) = \int_r^R E \cdot d\ell = \int_\rho^R \frac{\rho r}{3\varepsilon_0}\, dr = \frac{\rho}{3\,\varepsilon_0} \left(\frac{r^2}{2}\right)\Big|_{\rho = r}^{R}$$

$$= \frac{\rho}{3\varepsilon_0}\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

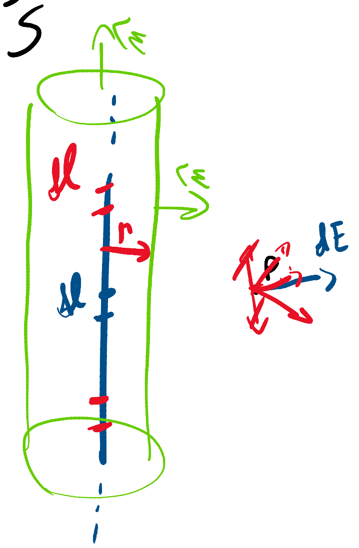
Ragioniamo sul filo:

Uso Gauss

$$\frac{Q}{\varepsilon_0} = \int_S \vec{E} \cdot \hat{n} \, dS$$

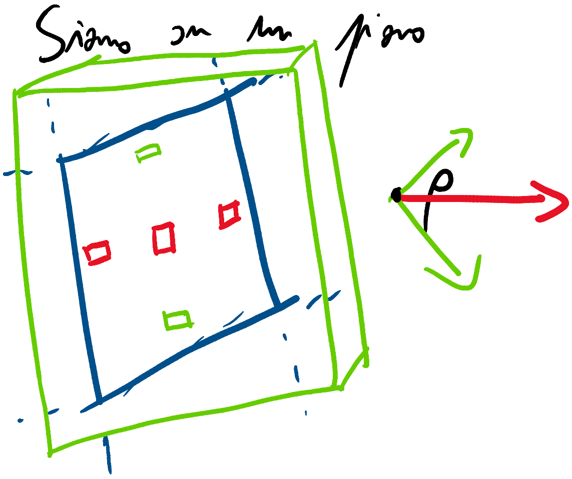
con $S =$ cilindro

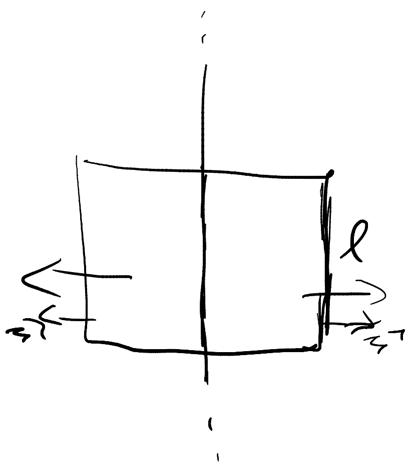
$\Rightarrow S = \underline{2\pi r h} + 2\pi r^2$



$$\frac{Q}{\varepsilon_0} = E \cdot 2\pi r h \Rightarrow E = \frac{\lambda L}{2\pi r h \varepsilon_0} = \frac{\lambda \cancel{h}}{2\pi r \varepsilon_0 \cancel{h}}$$

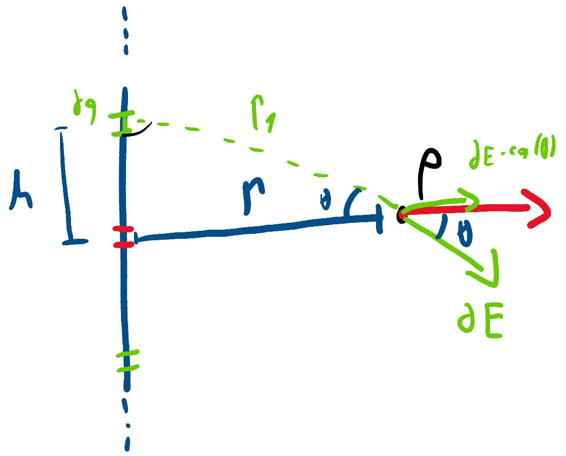
Siamo su un piano





$$\frac{Q}{\varepsilon_0} = \int_S E \cdot \hat{n} \, dS$$

$$\frac{Q}{\varepsilon_0} = E\,2l^2 \quad \Rightarrow E = \frac{Q}{2\varepsilon_0 l^2} = \frac{\sigma \cdot S}{\varepsilon_0 l^2} = \frac{\sigma l^2}{2\varepsilon_0 l^2} = \frac{\sigma}{2\varepsilon_0}$$

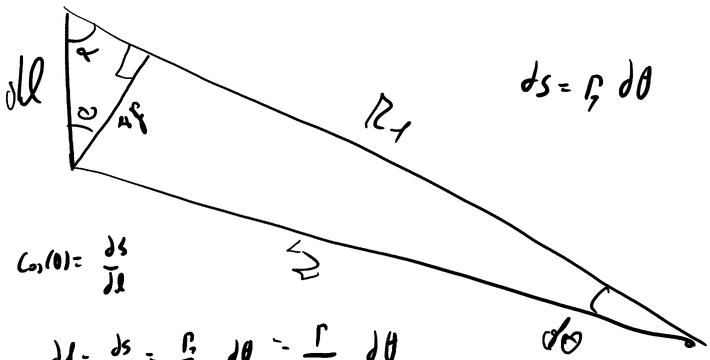


$$dE = \frac{dq}{4\pi\varepsilon_0 r_1^2}$$

$$dE \cdot \cos(\theta) = \frac{\lambda \cdot dL}{4\pi\varepsilon_0 r_1^2} \cdot \cos(\theta)$$

$$= \frac{\lambda \cdot dL \cdot \cos^3(\theta)}{4\pi\varepsilon_0 r^2} = \frac{\lambda \cdot d\theta \cos(\theta)}{4\pi\varepsilon_0 r}$$

$$\cos(\theta) = \frac{r}{r_1} \qquad \Rightarrow r_1 = \frac{r}{\cos(\theta)}$$



$$ds = r_1\, d\theta$$

$$\cos(\theta) = \frac{ds}{dl}$$

$$dl = \frac{ds}{\cos(\theta)} = \frac{r_1}{\cos(\theta)}\, d\theta = \frac{r}{\cos^2(\theta)}\, d\theta$$

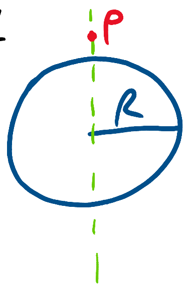
$$\Rightarrow E = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda \cos(\theta)}{4\pi\varepsilon_0 r}\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r} \cdot \left(\sin(\theta)\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

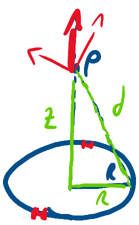
Esercitazione 3

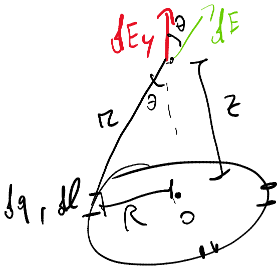
giovedì 30 novembre 2023 16:07

$\lambda = \frac{dQ}{dL}$

Es 1







$dE_y = dE \cos\theta$

$dE = \frac{dq}{4\pi\varepsilon_0 r^2} \Rightarrow$

$dE_y = \frac{\lambda\, dl}{4\pi\varepsilon_0 r^2} \cos\theta$

$\cos\theta = \frac{z}{r} = \frac{z}{(R^2+z^2)^{1/2}}$

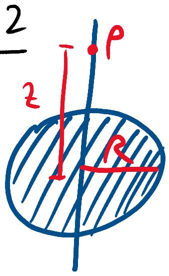
$dE_y = \frac{\lambda\, dl}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}\, z$

$E = \int_C$

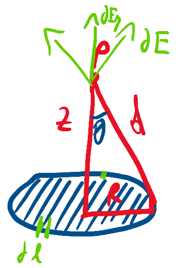
$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda\, dL}{4\pi\varepsilon_0 r^2}$

$\Rightarrow E = \frac{\overbrace{\lambda z\, 2\pi R}^{Q_{TOT}}}{4\pi\varepsilon_0 \, (R^2+z^2)^{3/2}} = \frac{Q_{TOT}\cdot z}{4\pi\varepsilon_0 \, (R^2+z^2)^{3/2}}$

Es 2



$\sigma = \frac{dQ}{dS}$



$r = \sqrt{z^2+R^2}$

$dE_y = dE \cdot \cos(\theta)$

$= dE \cdot \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}}$

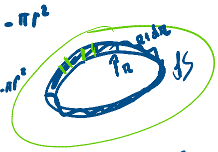
$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\sigma\, dS}{4\pi\varepsilon_0 (z^2+R^2)}$

$\pi(r+dr)^2 - \pi r^2$

$\pi(r^2+dr^2+2r\,dr) - \pi r^2$

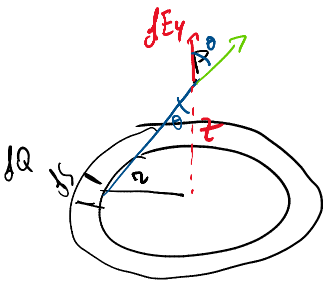
$= \pi(2r\,dr + \cancel{dr^2})$

$= \pi 2r\, dr$



$S = \pi r^2$

$dS = \pi \cdot 2r\, dr$



$\frac{dQ}{4\pi\varepsilon_0 d^2} = \frac{dQ}{4\pi\varepsilon_0 (r^2+z^2)}$

$dE_y = \frac{\sigma\, dS \cdot z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$

$$E_y = dE_y = \frac{\sigma \cdot 2\pi r\, dr \cdot z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$E_{TOT} = \int_0^R \frac{\sigma \cdot 2\pi z}{4\pi\varepsilon_0} \quad \frac{r}{(r^2+z^2)^{\frac{3}{2}}} dr$$

$$-(r^2+z^2)^{-\frac{1}{2}}$$

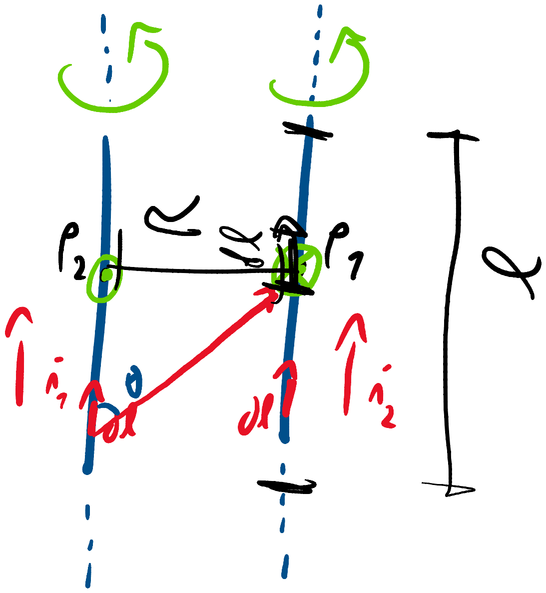
$$= \frac{\sigma \cdot z}{2\varepsilon_0} \left(-(r^2+z^2)^{-\frac{1}{2}}\right)\Big|_0^R$$

$$= \frac{\sigma \cdot z}{2\varepsilon_0} \left(-\left(\frac{1}{(R^2+z^2)^{\frac{1}{2}}} - \frac{1}{z}\right)\right)$$

$\blacksquare$

Esercitazione 4

giovedì 30 novembre 2023 17:20



$$B = \frac{\mu_0 \, i}{2\pi R}$$

$$dF = i \cdot (d\ell \wedge \bar{B}) = i_2 \cdot d\ell \cdot B$$

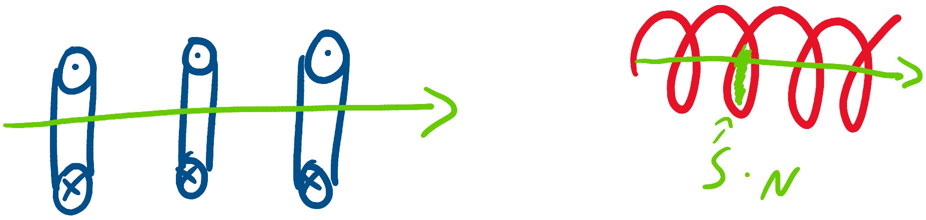
$$= i_2 \cdot \frac{\mu_0 \cdot i_1}{2\pi R} \cdot d\ell$$

$$F = \frac{i_1 \cdot i_2 \, \mu_0}{2\pi R} \cdot L$$

$$\frac{F}{L} = \text{densità di Forza}$$

Esercitazione 5

venerdì 1 dicembre 2023 16:00



$$B = \mu_0 \cdot n \cdot i = \mu_0 \frac{N i}{l}$$

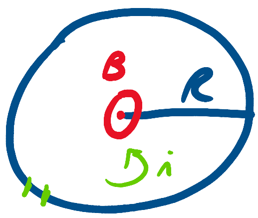
$$\phi_S(B) = \int_S \frac{\mu_0 N i}{l} \, dS = B \cdot N \cdot S = B \cdot n \cdot l \cdot S$$

$$= \mu_0 \cdot i \, n \cdot n \, l \cdot S$$

$$= \mu_0 \cdot n^2 l S i = L i$$

$$\Rightarrow L = \mu_0 n^2 l S$$

Esercizio



Calcolare camp magnetico

$$dB = \frac{\mu_0 i}{4\pi} \frac{dl \wedge \bar{r}}{r^3}$$

$$B = \int_\gamma \frac{\mu_0 \cdot i}{4\pi r^2} dl = \frac{\mu_0 i}{4\pi} \int_\gamma \frac{1}{r^2} dl$$

γ γ

$$= \frac{\mu_0\, i}{4\pi} \cdot \frac{1}{r^2} \cdot 2\pi r = \frac{\mu_0\, i}{2r}$$

Esercizio 2

$$\rho = \frac{A}{r}$$

Trovare E

Usiamo Gauss

Sia $r > R$

$$\int_S E\, ds = \frac{Q}{\varepsilon_0}$$

$$\rho = \frac{dQ}{dV}$$

$$dQ = \rho\, dV$$

$$Q = \int_V \rho\, dV$$

$$Q = \int_V \frac{A}{r}\, dV = \int_0^R \frac{A}{\cancel{r}} 4\pi r^{\cancel{2}}\, dr = A 4\pi \frac{r^2}{2} \Big|_0^R$$

$$= 4A\pi \frac{R^2}{2}$$

$$V = \frac{4}{3}\pi R^3$$

$$dV = 4 \cdot \pi \cdot 3R^2\, dR$$

$$V = \frac{}{3}\ldots$$

$$dV = \frac{4}{3} \cdot \pi \cdot 3R^2 \, dR$$

Se $r<R$:

$$\rho = \frac{dQ}{dV} \Rightarrow dQ = \rho \, dV$$

$$\Rightarrow Q = \int_V \rho \, dV = \int_V \frac{A}{r} dV$$

$$V = \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2$$

$$= \int_0^r \frac{A}{\cancel{r}} 4\pi r^{\cancel{2}} \, dr$$

$$= A 4\pi \left. \frac{r^2}{2} \right|_0^r$$

$$= 4A\pi \frac{r^2}{2} = 2A\pi r^2$$

$$E \cdot \overset{2}{\cancel{4}} \cancel{\pi} \cancel{r}^2 = \frac{\cancel{2} A \cancel{\pi} \cancel{r}^2}{\varepsilon_0} = \frac{A}{2\varepsilon_0}$$

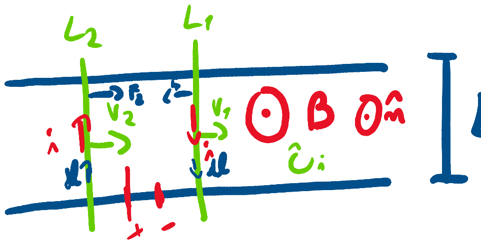
Appello Giugno 2023

venerdì 1 dicembre 2023 16:10

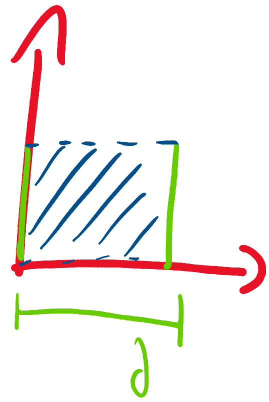
Es 2

$i = 0{,}24\ A$ $\quad B = 1{,}2\ T$ $\quad V_1 = 10\ m/s$

$V_2 = 5\ m/s$



$L = 40\ cm = 0{,}4\ m$



$x_1(t) = d + V_1 t$

$x_2(t) = V_2 t$

$x_1 - x_2 = d + V_1 t - V_2 t$

$(x_1 - x_2) L = (d + V_1 t - V_2 t) \cdot L$

1) Resistenza di ciascuna barretta:

$$\Delta V = R \cdot i$$

$$\Delta V = \xi = -\frac{d}{dt}\phi_s(B)$$

Ora

$$\phi_s(B) = \int_S \bar{B} \cdot \hat{n}\, dS$$

$$= B \cdot (d + V_1 t - V_2 t) \cdot L$$

$$-\frac{d}{dt}\phi(B) = -BL\,(V_1 - V_2)$$

$$\xi = -BL\,(V_1 - V_2)$$ $\Rightarrow$ La corrente gira in senso orario

$$R_{eq} = \frac{\xi}{i}$$

$\Rightarrow R = \dfrac{R_{eq}}{2}$ Perché $R_{eq} = R_1 + R_2$ siccome sono in Serie

$$R = -\frac{BL(V_1 - V_2)}{i} \cdot \frac{1}{2} = 5$$

2) La Forza agente su ciascuna sbarra:

$$dF = i \cdot dl \wedge \vec{B}$$

$$\Rightarrow F_2 = \int_{L_2} iB\, dl = iB \cdot L_2$$

$$F_1 = \int_{L_1} iB\, dl = iB\, L_1$$

3): La carica che ha percorso il circuito dopo $t = 10$,

$$i = \frac{dQ}{dt}$$

$$dQ = i\, dt$$

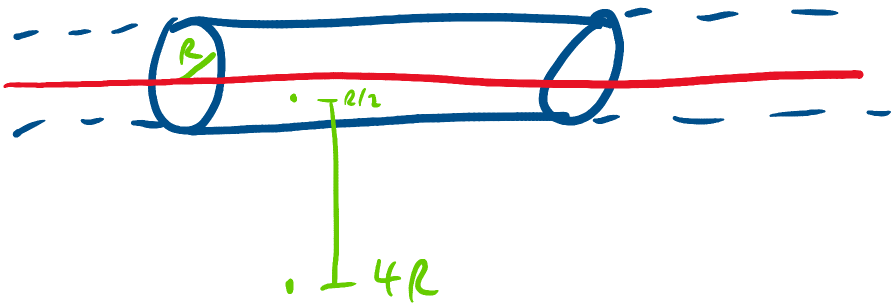
$$\Rightarrow Q = \int_0^{10} i\, dt = i \cdot 10 = 2{,}4\ C$$

$\blacksquare$

Es 3

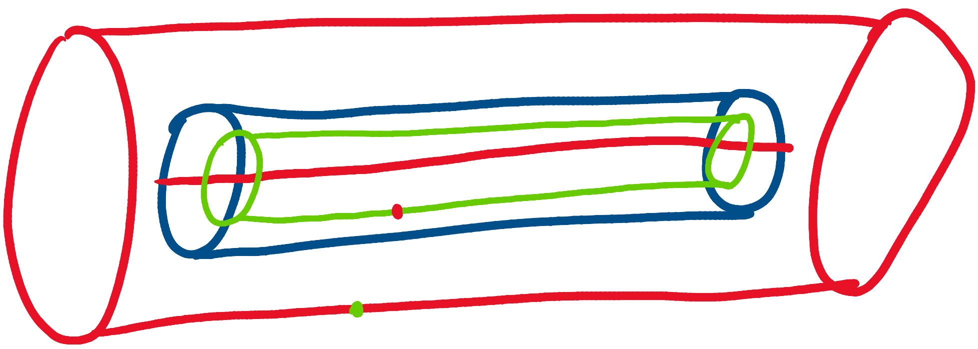
$R = 1\,m$ $\rho = 10^{-6}\,C/m^3$ $q = 1{,}6 \cdot 10^{-19}\,C$

$m = 10^{-19}\,kg$



a) Accelerazione nel punto $R_1 = R/2$:

Usiamo Gauss



$$\int_S \bar{E} \cdot \hat{n}\, dS = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \int_S \bar{E}\, dS = \frac{Q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0}$$

$$E \int_S dS = \frac{\rho \cdot V}{\varepsilon_0}$$

$V = \pi r^2 \cdot h$

ρV

$$E \cdot \cancel{2}\pi \frac{R}{\cancel{2}} \cdot h = \frac{\rho \cdot V}{\varepsilon_0}$$ $V = \pi r^2 \cdot h$

$\Rightarrow$

$$E = \frac{\rho \cdot V}{\varepsilon_0 \ \pi \cdot R \cdot h} = \frac{\rho \cdot \cancel{\pi} \left(\frac{R}{2}\right)^2 \cdot \cancel{h}}{\varepsilon_0 \cancel{\pi} R \cancel{h}} =$$

$$= \frac{\rho R^2}{4 \varepsilon_0 R} = \frac{\rho R}{4 \varepsilon_0}$$

$\Rightarrow$ $F = E \cdot q = \frac{\rho R}{4 \varepsilon_0}$

$F = m \cdot a \quad \Rightarrow \quad a = \frac{F}{m} = \frac{\rho \cdot R}{4 \varepsilon_0 m}$

b)

$$\int_S E \, ds = \frac{Q}{\varepsilon_0}$$

$V = \pi (R)^2 \cdot h$

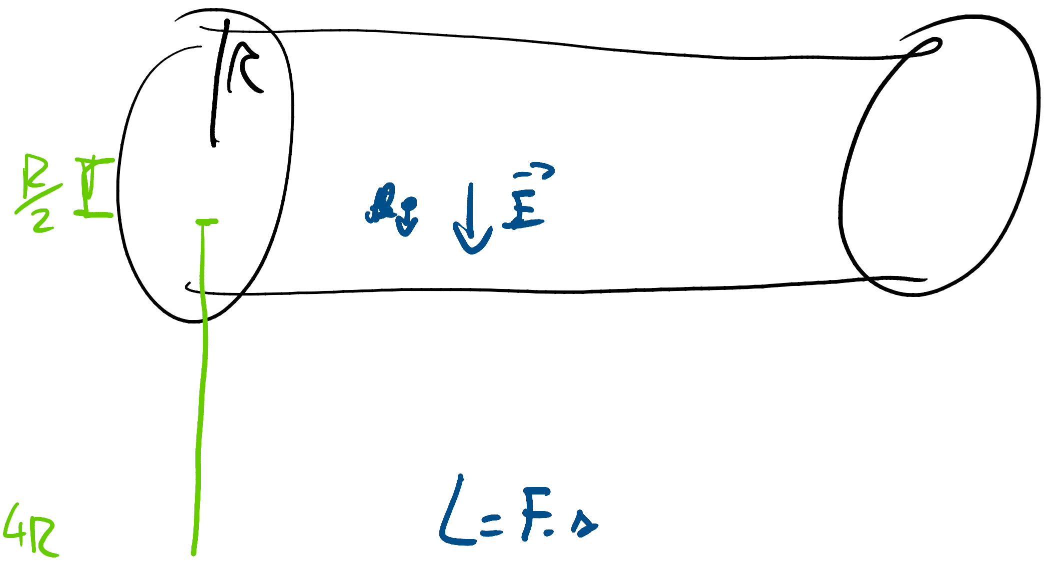
$$E \cdot 2 \pi 4R h = \frac{\rho V}{\varepsilon_0}$$

$\Rightarrow$

$$E = \frac{\rho \cdot \cancel{\pi} \cdot R^{\cancel{2}} \cdot \cancel{h}}{} = \frac{\rho R}{}$$

$$E = \frac{\rho \cdot \cancel{\pi} \cdot R^2 \cdot \cancel{h}}{\varepsilon_0 \, 2 \cancel{\pi} \, 4 \cancel{R} \, \cancel{h}} = \frac{\rho R}{8 \varepsilon_0}$$

c): Velocità in arrivo in $R_2 = 4R$:



$$L = F \cdot s$$

$$L = \int_{\gamma} F \cdot d\ell = \int_{\gamma} E q \, d\ell$$

$$= \int_{R/2}^{R} + \int_{R}^{4R}$$

$r < R$

$$\int_S E \, ds = \frac{Q}{\varepsilon_0}$$

$$E \cdot 2\cancel{\pi} \cancel{r} \cdot \cancel{h} = \frac{Q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0} = \frac{\rho \cdot \cancel{\pi} r^{\cancel{2}} \cancel{h}}{\varepsilon_0}$$

$$E = \frac{\rho r}{2\varepsilon_0}$$

Si $r > R$

$$\int_S E \, ds = \frac{Q}{\varepsilon_0}$$

$\Rightarrow$

$$E \cdot 2\cancel{\pi} r \cancel{h} = \frac{\rho \cancel{\pi} R^2 \cancel{h}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho \cdot R^2}{2\varepsilon_0} \cdot \frac{1}{r}$$

$$\Rightarrow L = \int_{R/2}^{R} q \frac{\rho r}{2\varepsilon_0} \, dr + \int_{R}^{4R} \frac{\rho R^2}{2\varepsilon_0} q \cdot \frac{1}{r} \, dr$$

$$L = q \cdot \rho \quad r^2 \Big|^{R} + \rho R^2 q \quad \ln(r) \Big|^{4R}$$

$$L = \frac{q \cdot \rho}{2\varepsilon_0} \cdot \frac{r^2}{2}\Bigg|_{R/2}^{R} + \frac{\rho R^2 q}{2\varepsilon_0} \cdot \ln(r)\Bigg|_{R}^{4R}$$

$$\underbrace{\text{Energ Cin Fin}}_{\frac{1}{2} m v_f^2} - \underbrace{\text{Energia Cin Iniz}}_{0} = L$$

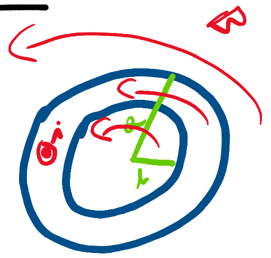
$$v_f = \sqrt{\frac{2L}{m}}$$

$\blacksquare$

Gennaio 2023

sabato 2 dicembre 2023 15:12

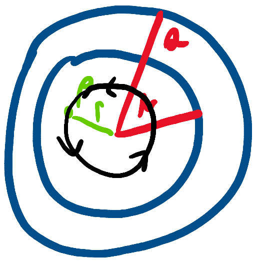
Es 1



Usiamo Ampère

$$\oint_\gamma \bar{B}\, dl = \sum \mu_0 \cdot i$$

1) $r < b$



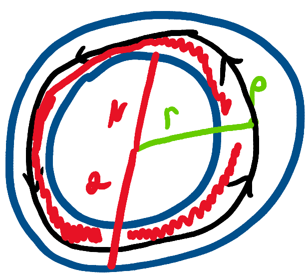
$$\oint_\gamma \bar{B}\cdot d\vec{l} = \sum \mu_0 \cdot i_{conc} = 0$$

$\Longleftrightarrow$ Dato che B e dl sono non perpendicolari

$\Rightarrow$ B deve essere nullo

$\Rightarrow B = 0$ se $r < b$

2) $b \le r \le a$



$$J = \frac{di}{dS} = \frac{i}{S} = \frac{i}{\pi(a^2 - b^2)}$$

$$\oint \bar{B}\cdot d\vec{l} = \sum \mu_0 \, i_{conc}$$

$$\oint_\gamma \bar{B}\cdot d\bar{l} = \sum \mu_0 \, i_{conc}$$

$\Rightarrow$

$$i_{conc} = J \cdot \pi (r^2 - b^2) = \frac{i}{\pi (a^2 - b^2)} \cdot \pi (r^2 - b^2)$$

$$\Rightarrow i_{conc} = \frac{i}{a^2 - b^2} \cdot (r^2 - b^2)$$

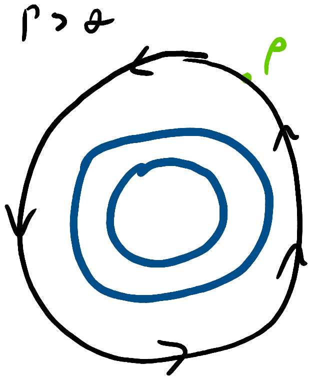
$$\oint_\gamma \bar{B} \cdot d\vec{l} = \mu_0 \cdot \frac{i}{a^2 - b^2} (r^2 - b^2)$$

Poiché è uniforme B

$$B \cdot 2\pi r = \mu_0 \frac{i}{a^2 - b^2} \cdot (r^2 - b^2)$$

$$\Rightarrow B = \frac{\mu_0 \, i \, (r^2 - b^2)}{(a^2 - b^2) \, 2\pi r}$$

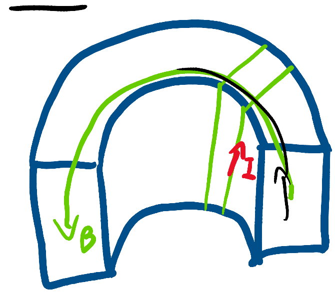
3) $r > a$

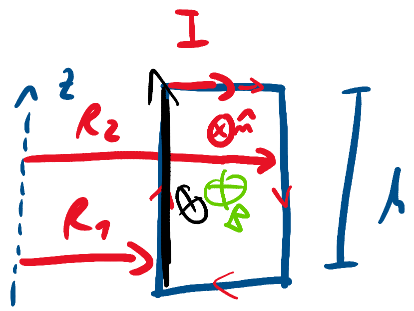


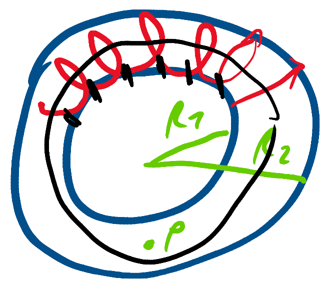
$$B \cdot 2\pi r = \mu_0 \, i \quad \Rightarrow \quad B = \frac{\mu_0 \, i}{2\pi r}$$

$\blacksquare$

Es 2







Ricavare B in funzione di r

Caso 1: $r < R_1$

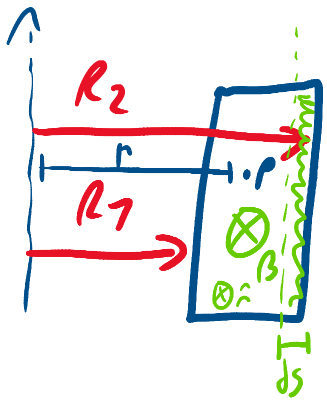
Usiamo Ampère

$$\oint_\gamma \bar{B}\, d\ell = \sum \mu_0 i_{conc} = 0$$

Come prima si ha $B = 0$

Caso 2 $: R_1 \leq r \leq R_2$

$$\oint_\gamma \bar{B} \cdot d\vec{\ell} = \sum \mu_0 i_{conc}$$



ds = h dr

$$\Rightarrow B \cdot 2\pi r = \mu_0 N \cdot I$$

$$\Rightarrow B = \frac{\mu_0 \cdot N \cdot I}{2\pi r}$$

Caso 3 $r > R_2$ $\Rightarrow$ Siccome le I sono a due a due opposte

$\Rightarrow B = 0$

Trovare L:

$$\phi(B) = L\, i$$

$$\int_S \bar{B} \cdot \hat{n}\, ds = L \cdot I$$

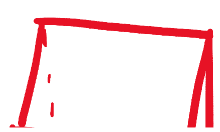
$$\int_S \bar{B}\, ds = L \cdot I \quad \Rightarrow \quad \int_{R_1}^{R_2} B h\, dr = \int_{R_1}^{R_2} \frac{\mu_0 N \cdot I}{2\pi} \cdot h \cdot \frac{1}{r}\, dr$$

$$= \frac{\mu_0 \cdot N \cdot I \cdot h}{2\pi} \cdot \ln\left(\frac{R_2}{R_1}\right)$$

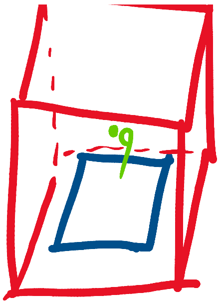
$$\Rightarrow L = \frac{\mu_0 \cdot N \cdot h}{2\pi} \cdot \ln\left(\frac{R_2}{R_1}\right)$$

Es 3

$$\oint_S \bar{E}\, ds$$



$\oint_S \bar{E} \ldots$



Por Gauss:

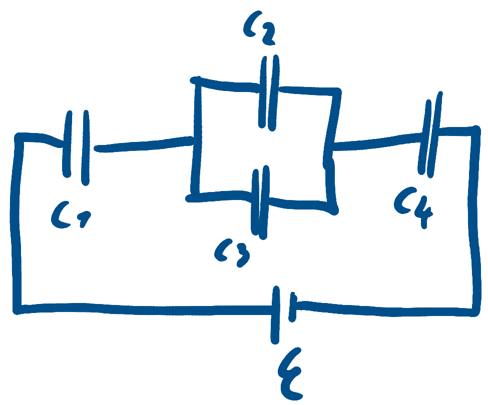
$$\oint_S \bar{E}\, ds = \frac{Q}{\varepsilon_0} \xRightarrow{\text{Dividindo por 6}} \oint_S \bar{E}\, ds = \frac{Q}{\varepsilon_0} \cdot \frac{1}{6}$$

$\blacksquare$

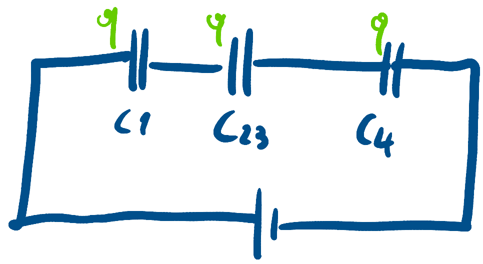
Febbraio 2023

sabato 2 dicembre 2023 16:10

Es 3



a) Capacità equivalente:



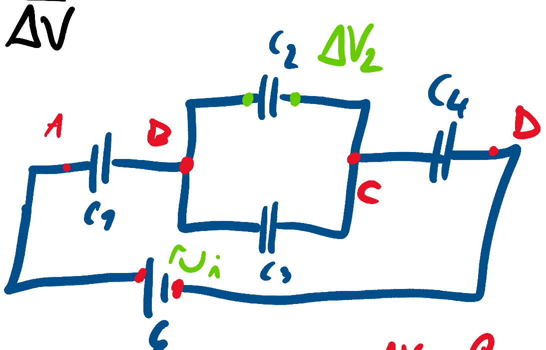
$C_{23} = C_2 + C_3$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}$$

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}\right)^{-1}$$

b):

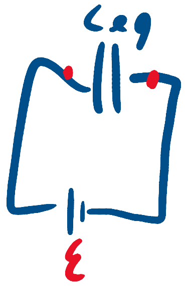
$$C_{apac} = \frac{Q}{\Delta V}$$



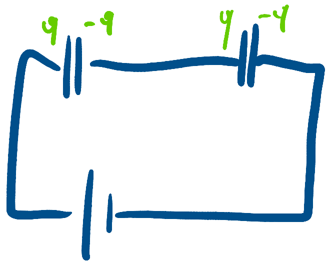
$\Delta V_1 = Q_{eq} \cdot \frac{1}{C_1}$

$\Delta V_4 = Q_{eq} \cdot \frac{1}{C_4}$

$$\varepsilon = V_D - V_A = \underbrace{(V_B - V_A)}_{\Delta V_1 = Q_{eq}\cdot\frac{1}{C_1}} + (V_C - V_B) + \underbrace{(V_D - V_C)}_{\Delta V_4 = Q_{eq}\cdot\frac{1}{C_4}}$$



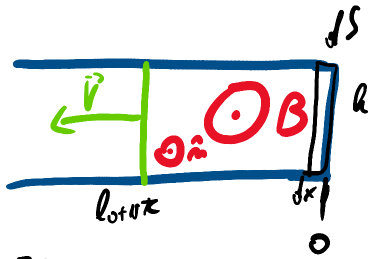
$$Q_{eq} = C_{eq} \cdot \varepsilon$$



$$Q_2 = \underbrace{\left(\varepsilon - (\Delta V_1 + \Delta V_4)\right)}_{\Delta V_2 = \Delta V_3} \cdot C_2$$

c): $\Delta V_4 = Q_{eq} \cdot \frac{1}{C_4}$

Ej 1



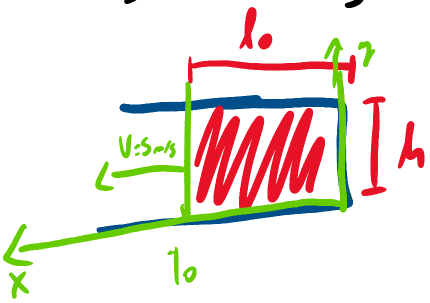
$$dS = l\,dx$$

a) Determinar F.E.M

$$\varepsilon = -\frac{d\,\phi(B)}{dt}$$

$$\phi(B) = \int \bar{B}\cdot\hat{n}\,dS = \int \bar{B}\,dS = \int^{l_o + vt} B \cdot l\,dx$$

$$\phi(B) = \int_S \bar{B}\cdot\hat{n}\, ds = \int_S \bar{B}\, ds = \int_0 B\cdot \ell\, dx$$



$$x(t) = S_0 + V\cdot t = l_0 + V\cdot t$$

$$S_{sup}(t) = (l_0 + V\cdot t)\cdot h$$

$\Rightarrow$

$$\phi(B) = B\cdot h\,(l_0 + V\cdot t)$$

$$\frac{d}{dt}\phi(B) = B\cdot h\cdot V$$

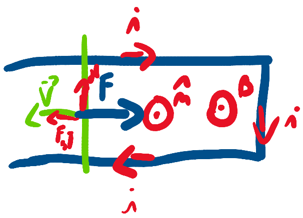
$$\varepsilon = -B\,h\,V_{el}$$

2)

Usiamo Ohm

$$\Delta V = R\cdot i \qquad \Rightarrow \quad i = \frac{\Delta V}{R} = \frac{-B\,h\,V_{el}}{R}$$

3)



Uso II Legge Laplace:

$$dF = i\; d\ell \wedge \bar{B}$$

$$F = \int_0^h iB\,dl = iBh$$

4) Potenza

$$W = \frac{dL}{dt} = \frac{d(F\cdot S)}{dt}$$

$\Rightarrow$

$$F\cdot S = iBh\,(l_0 + vt)$$

$$\frac{d\,F\cdot S}{dT} = iBhv \cdot (-1)$$

Potenza

Es 2

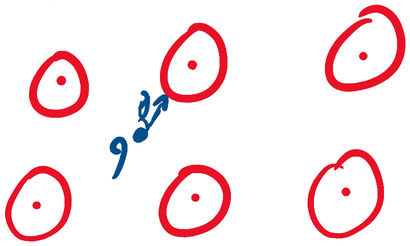
e

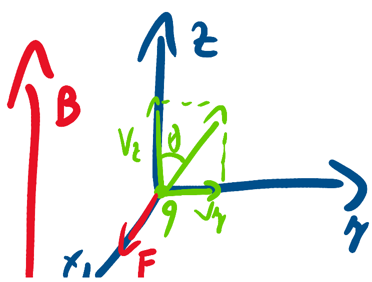
ΔV

B

$\theta = \frac{\pi}{3}$

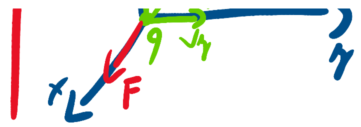
a) Periodo di rotazione





$V_y = V \sin(\theta)$

$V_z = V \cos(\theta)$



$V_z = V\cos(\theta)$

$F = q\,\vec{v}\wedge\vec{B}$

$a = \frac{F}{m}$, $a = W\cdot V_y \Rightarrow w = \frac{a}{V_y} \Rightarrow \frac{2\pi}{T} = \frac{a}{V_y} \Rightarrow T = \frac{2\pi V_y}{a}$

$K_{fin} - K_{in} = \frac{1}{2}mV_f^2 - \cancel{\frac{1}{2}mV_i^2}^{\sim 0} = U_i - U_f$

$= -\Delta U = -\Delta V\cdot q$

$\frac{1}{2}mV_f^2 = -\Delta V\, q$

$\Rightarrow V_f = \sqrt{\frac{\Delta V q\cdot 2}{m}}$

2)

$w\cdot r = V$, $w\cdot r^2 = a$

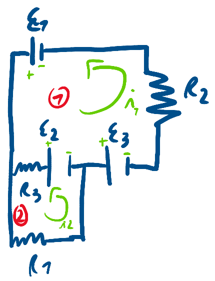
$r = \frac{V}{w}$

3) $p_{asso} = V_z\cdot T$

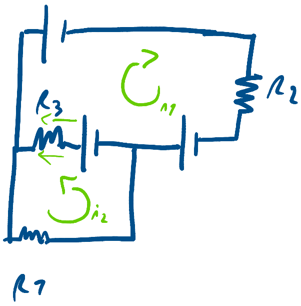
Settembre 2023

domenica 3 dicembre 2023 15:55

Es 3



Dopo i conti $\Downarrow$ $\Rightarrow$



II Legge di Kirchhoff:

$$
\begin{cases}
-\mathcal{E}_3 - i_1 R_2 + \mathcal{E}_1 - R_3(i_1 - i_2) - \mathcal{E}_2 = 0 & \text{maglia 1} \\
\mathcal{E}_2 - R_3(i_2 - i_1) - R_1 i_2 = 0 & \text{maglia 2}
\end{cases}
$$

$$
\Rightarrow \begin{cases}
-\mathcal{E}_3 - i_1 R_2 + \mathcal{E}_1 - R_3 i_1 + R_3 i_2 - \mathcal{E}_2 = 0 \\
\mathcal{E}_2 - R_3 i_2 + R_3 i_1 - R_1 i_2 = 0
\end{cases}
\qquad
\begin{cases}
-4 - 50 i_1 + 6 - 50 i_1 + 50 i_2 - 5 = 0 \\
5 - 50 i_2 + 50 i_1 - 100 i_2 = 0
\end{cases}
$$

$$
\Rightarrow \begin{cases}
i_1(-R_2 - R_3) - \mathcal{E}_3 + \mathcal{E}_1 - \mathcal{E}_2 + R_3 i_2 = 0 \\
\mathcal{E}_2 - R_3 i_2 + R_3 i_1 - R_1 i_2 = 0
\end{cases}
\qquad
\begin{cases}
-100 i_1 + 50 i_2 = 3 \\
50 i_1 - 150 i_2 = -5
\end{cases}
\Rightarrow
\begin{cases}
-100 i_1 + 50 i_2 = 3 \\
100 i_1 - 300 i_2 = -10
\end{cases}
$$

$$
\Rightarrow \begin{cases}
i_1 = \dfrac{-\mathcal{E}_3 + \mathcal{E}_1 - \mathcal{E}_2 + R_3 i_2}{R_2 + R_3} \\
\mathcal{E}_2 - R_3 i_2 + R_3 \left(\dfrac{-\mathcal{E}_3 + \mathcal{E}_1 - \mathcal{E}_2 + R_3 i_2}{R_2 + R_3} \right) - R_1 i_2 = 0
\end{cases}
\qquad
\begin{cases}
-250 i_2 = -7 \Rightarrow i_2 = \dfrac{7}{250} \\
-100 i_1 + 50 \left(\dfrac{7}{250}\right) = 3
\end{cases}
\Rightarrow
\begin{cases}
i_2 = \dfrac{7}{250} \\
i_1 = \dfrac{\left(3 - \frac{7}{5}\right)}{-100} = -\dfrac{2}{125}
\end{cases}
$$

$$
\Rightarrow \begin{cases}
\text{''} \\
\mathcal{E}_2 - R_3 i_2 - \dfrac{\mathcal{E}_3 R_3}{R_2 + R_3} + \dfrac{\mathcal{E}_1 R_3}{R_2 + R_3} - \dfrac{\mathcal{E}_2 R_3}{R_2 + R_3} + \dfrac{R_3^2}{R_2 + R_3} i_2 - R_1 i_2 = 0
\end{cases}
$$

$$
\Rightarrow \begin{cases}
\text{''} \\
i_2 \left(-R_3 + \dfrac{R_3^2}{R_2 + R_3} - R_1 \right) = -\mathcal{E}_2 + \dfrac{\mathcal{E}_3 R_3}{R_2 + R_3} - \dfrac{\mathcal{E}_1 R_3}{R_2 + R_3} + \dfrac{\mathcal{E}_2 R_3}{R_2 + R_3}
\end{cases}
$$

$$
\Rightarrow i_2 = \frac{-\mathcal{E}_2 + \dfrac{\mathcal{E}_3 R_3}{R_2 + R_3} - \dfrac{\mathcal{E}_1 R_3}{R_2 + R_3} + \dfrac{\mathcal{E}_2 R_3}{R_2 + R_3}}{-R_3 + \dfrac{R_3^2}{R_2 + R_3} - R_1} \underset{\text{sostituendo i valori}}{=} \dots = \frac{7}{250}
$$

$i_1 = -\frac{2}{125}$

- in R_1 scorre corrente i_2 in senso anti-orario
- in R_2 scorre corrente i_1 in senso orario
- in R_3 scorre corrente $(i_1 + i_2)$ senso sinistra

Calcoliamo ora le potenze:

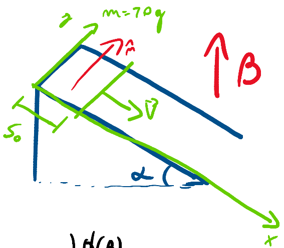
$W_1 = i_2^2 \cdot R_1$

$W_2 = i_1^2 \cdot R_2$

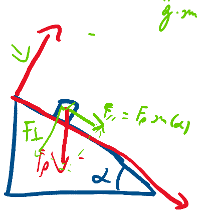
$W_3 = (i_1 + i_2)^2 \cdot R_3$

Es 1

$l = 10\,cm$ $R = 10\,\Omega$



$x(t) = S_0 + \cancel{V_0 t} + \frac{1}{2} a t^2$, $a = g \cdot \sin(\alpha)$



1) $\xi = -\frac{d\phi(B)}{dt}$, i_{indotta} ,

$$\phi(B) = \int_S \bar{B} \cdot \hat{n} \cdot dS = \int_S B \cdot \cos(\alpha)\, dS$$

$$= B \cdot \cos(\alpha) \int_S dS$$

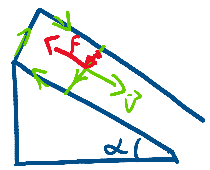
$$= B \cdot \cos(\alpha) \cdot \left(S_0 + \cancel{V_0} t + \frac{1}{2} g \sin(\alpha) \cdot t^2\right) l$$

$$\frac{d\phi(B)}{dt} = B \cdot \cos(\alpha) \cdot g \cdot \sin(\alpha) \cdot t \cdot l$$

$$\xi = -B \cos(\alpha) \underbrace{\sin(\alpha) \cdot g}_{a} \cdot t \cdot l = -B \cos(\alpha) \cdot \underbrace{V(t)}_{a \cdot t} \cdot l$$

$$i_{ind} = \frac{\xi}{R} = -\frac{B \cos(\alpha)\, l\, V(t)}{R}$$

2):



$$F = \underset{\text{indotta}}{i}\, dl \wedge B = i B\, dl \sin\left(\tfrac{\pi}{2}\right) = i B\, dl = i B l$$

$$a = \frac{F}{m} = \frac{iBl}{m} = -\frac{B^2 \cos(\alpha)\, l^2\, V(t)}{R}$$

$$a_{TOT} = -\frac{B^2 \cos(\alpha)\, l^2\, V(t)}{R} + \underbrace{g \sin(\alpha)}_{B} \qquad A = -\frac{B^2 \cos(\alpha)\, l^2}{R}$$

$$\dot V(t) = A \cdot V(t) + B \Rightarrow \frac{dV}{A \cdot V(t) + B} = dt \Rightarrow \int_0^V \frac{1}{A \tilde V(t) + B}\, d\tilde V = \int_0^t dT$$

$$\frac{dV}{dt} = A V(t) + B$$

$$\Rightarrow \left.\frac{\ln(A V(t) + B)}{A}\right|_0^V = t$$

$$\Rightarrow \frac{1}{A}\Big(\ln(A \cdot V + B) - \ln(B)\Big) = T$$

$$\Rightarrow \ln\left(\frac{AV+B}{B}\right) = At$$

$$\Rightarrow \frac{AV+B}{B} = e^{At}$$

$$\Rightarrow AV + B = B e^{At}$$

$$\Rightarrow V = \frac{B e^{At} - B}{A} \xrightarrow{t \to \infty} 0$$

$$\boxed{\begin{aligned} & y' = a(x)\, y + b(x) \\ & y = e^{+A(x)} \int e^{-A(x)}\, b(x)\, dx \end{aligned}}$$

$$\Rightarrow V = e^{A \cdot t} \int e^{-A \cdot t} B\, dt = e^{At} B \int e^{-At}\, dT$$

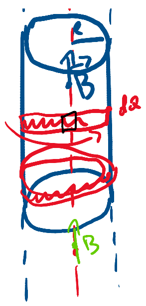
$$= e^{At} B \left(e^{-At} \cdot \left(-\frac{1}{A}\right)\right) =$$

$$= -\frac{B}{A}$$

Es 2

$$\sigma = \frac{dQ}{dS}$$

dQ :



$\sigma = \frac{}{\partial S}$

$\frac{dQ}{dt} = i$

$dS = r\, d\alpha\, dh$

Trovare B :

$$dB = \frac{\mu_0 \cdot i}{4\pi} \frac{dl \wedge r}{r^3}$$

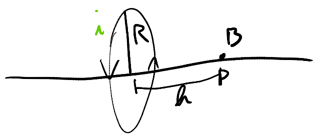
Troviamo i :

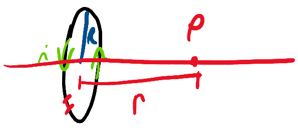
$$\frac{dQ}{dt} = i \qquad , \quad \sigma = \frac{dQ}{dS} \Rightarrow dQ = \sigma\, dS$$

$$\boxed{dS = r\, d\alpha\, dh}$$

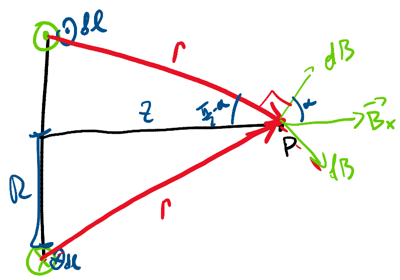
$$dQ = \sigma\, d\alpha\, r\, dh$$

$$i = \frac{\sigma\, d\alpha\, r\, dh}{dt} = \sigma\, \omega\, r\, dh$$





$$dB = \frac{\mu_0 i}{4\pi} \frac{dl \wedge \bar{r}}{r^3}$$



$$\cos(\alpha) = \frac{R}{r} = \frac{R}{\sqrt{R^2+z^2}}$$

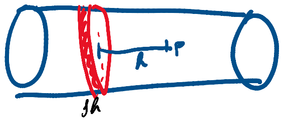
$$dB_x = dB \cos(\alpha) = dB \cdot \frac{R}{\sqrt{R^2+z^2}} = \frac{\mu_0 i}{4\pi} \cdot \frac{dl}{r^2} \cdot \frac{R}{\sqrt{R^2+z^2}}$$

$\Rightarrow$

$$B = \frac{\mu_0 \cdot i}{4\pi} \cdot \frac{R}{\left(z^2+R^2\right)^{3/2}} \int_0^{2\pi R} dl$$

$$= \frac{\mu_0 \cdot i}{} \quad \frac{R}{} \quad 2\pi R = \frac{\mu_0\, i\, R^2}{}$$

$$= \frac{\mu_0 \cdot i}{2\cancel{4\pi}} \cdot \frac{R}{(z^2+R^2)^{3/2}} \cdot \cancel{2\pi} R = \frac{\mu_0\, i\, R^2}{2\,(z^2+R^2)^{3/2}}$$



Ora $$B_{cilindrino} = \frac{\mu_0\, i\, R^2}{2\,(z^2+R^2)^{3/2}} = \frac{\mu_0\, R^2}{2\,(z^2+R^2)^{3/2}} \cdot \sigma \omega R \, dz$$

$$B_{TOT} = \int_{-\infty}^{\infty} B_{cilindr} = \frac{\mu_0 R^3 \sigma \omega}{2} \int_{-\infty}^{\infty} \frac{1}{(z^2+R^2)^{3/2}} \, dz$$

Esercizi da rivedere

domenica 14 gennaio 2024 17:07

APPELLO 10/09/2021 : ES 1,

: Es 2

// 16/11/2021

Lavan

// 18/02/2022 : Es 1, 2, 3

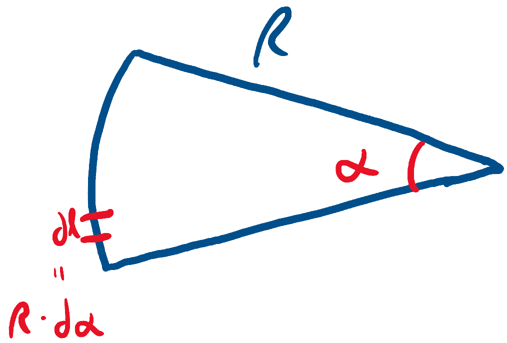
// 15/03/2022 : Es 2

// 30/06/2022 : Es 1, 2, 3

Appello 10/09/2021

venerdì 12 gennaio 2024 15:51

Es 1



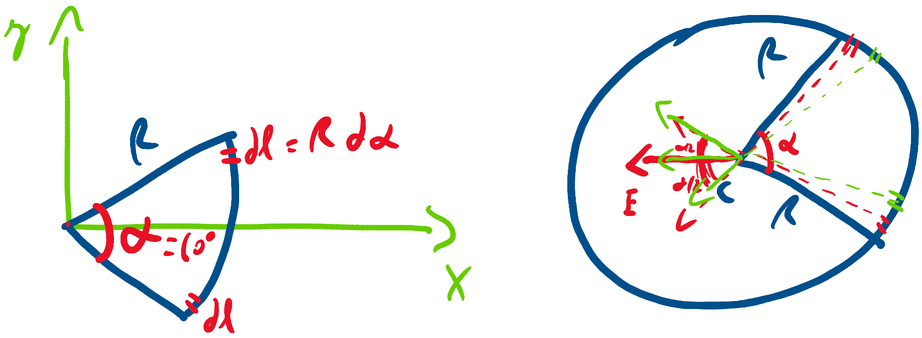
$R = 6\ cm$

$\alpha = 60° = \frac{60 \cdot \pi}{180} = \frac{1}{3}\pi$

$\lambda = 14\ \frac{nC}{m}$

Calcolare E

Sol



OSS: La risultante è solo sull'asse delle x

$\Rightarrow dE_x = dE \cos(\alpha)$ e $\alpha = \frac{\pi}{3} \Rightarrow \frac{\alpha}{2} = \frac{\pi}{6}$

Inoltre $\lambda = \frac{dQ}{dl} \Rightarrow dQ = \lambda\, dl = \lambda R\, d\alpha$

$$dE_x = dE \cos(\alpha) = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r^2} \cdot \cos(\alpha) =$$

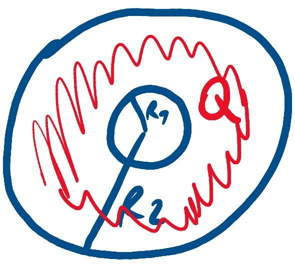
$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dl}{R^2} \cos(\alpha) = \frac{1}{4\pi\varepsilon_0} \frac{\lambda \cancel{R}}{R^{\cancel{2}}} \cos(\alpha)\, d\alpha$$

$\Rightarrow$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(\alpha)\, d\alpha$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R} \cdot \sin(\alpha) \Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \dots = {}^{*}\text{BOH UN NUMERO}$$

Es 2



$R_2 = 9\ cm$

$Q = 18\ \mu C$

a) Determinare il campo elettrico per

$$r > R_2$$

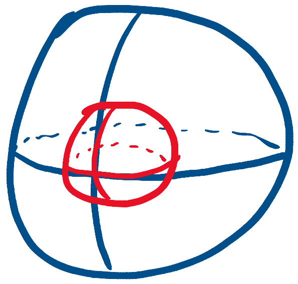
$$R_1 \leq r \leq R_2$$

$$r < R_1$$

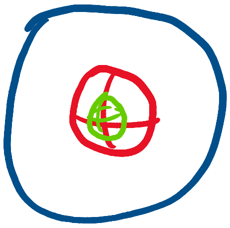
b) Il Potenziale Elettrostatico sulla superficie esterna della sfera di raggio R

Sol

a)



Se $r < R_1$



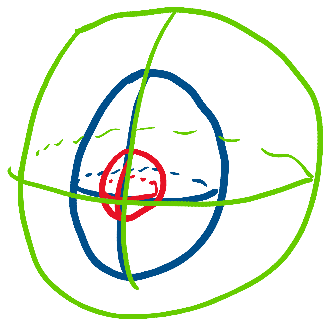
Per Gauss

$$\phi_s(E) = \int_s \bar{E}\,\hat{n}\,ds = \frac{Q_{int}}{\varepsilon_0}$$

Ma la cavità è cava $\Rightarrow$ Quindi per $r < R_1$ si ha
$Q_{int} = 0$

$\Rightarrow$
$E = 0$ se $r < R_1$

. Se $r > R_2$



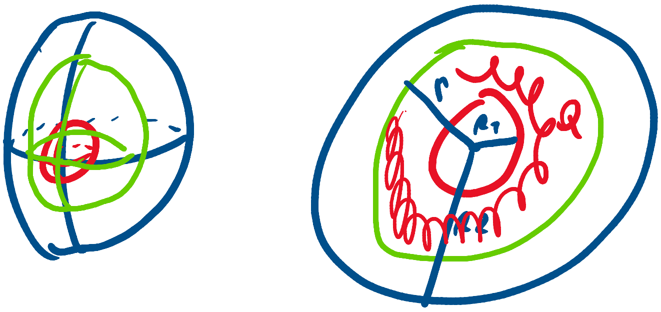
$$\int_S E \, ds = \frac{Q}{\varepsilon_0}$$

$\Rightarrow$

$$E \cdot S_{sfera} = \frac{Q}{\varepsilon_0} \Rightarrow E \, 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

- Se $R_1 \leq r \leq R_2$



$$\int_S E \, ds = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$E \, 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0}$$

Sappiamo che $\rho = \frac{\partial Q}{\partial V} \Rightarrow \partial Q = \rho \, \partial V$

della sfera che interessa a noi

$$\Rightarrow Q = \int_V \rho \, \partial V$$

Poiché il volume di una sfera è dato da

$$V = \frac{4}{3}\pi r^3$$

$\Rightarrow$

$$\partial V = \frac{4}{3}\pi \, 3r^2 \, \partial r$$

$\Rightarrow$

$$Q_{int} = \int_{R_1}^{r} \rho \, 4\pi r^2 \, \partial r$$

$\Rightarrow$

$$Q_{int} = \rho 4\pi \left. \frac{r^3}{3} \right|_{R_1}^{r} = \rho 4\pi \left(\frac{r^3}{3} - \frac{R_1^3}{3} \right) = \rho 4\pi \frac{r^3 - R_1^3}{3}$$

$$= \rho \frac{4}{3}\pi \left(r^3 - R_1^3 \right)$$

Quindi

$$E \, 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi \left(r^3 - R_1^3 \right)}{\varepsilon_0}$$

$\Rightarrow$

$$E = \frac{\rho}{\varepsilon_0} \cdot \frac{4}{3}\pi \left(r^3 - R_1^3 \right) \cdot \frac{1}{4\pi r^2}$$

Dobbiamo ancora sistemare ρ

Sappiamo che

$$\rho = \frac{\partial Q}{\partial V}$$

ma anche

Q che interessa a noi

ma anche

$$\rho = \frac{Q_{TOT}}{V} \qquad \left[\text{o comunque } \rho = \frac{dQ}{dV} \right]$$

Volume che contiene la carica

è

Volume che prendiamo in modo adeguato

Nel nostro caso

$$V = V_{sfera_{R_2}} - V_{sfera_{R_1}} = \frac{4}{3}\pi \left(R_2^3 - R_1^3\right)$$

Allora

$$E = \frac{Q_{TOT}}{V} \cdot \frac{1}{\varepsilon_0} \cdot \frac{4}{3}\pi \left(r^3 - R_1^3\right) \cdot \frac{1}{4\pi r^2}$$

$$= \frac{Q}{\cancel{\frac{4}{3}\pi} \left(R_2^3 - R_1^3\right)} \cdot \frac{1}{\varepsilon_0} \cdot \cancel{\frac{4}{3}\pi} \left(r^3 - R_1^3\right) \cdot \frac{1}{4\pi r^2} =$$

$$= \frac{Q\left(r^3 - R_1^3\right)}{\varepsilon_0 \left(R_2^3 - R_1^3\right) 4\pi r^2}$$

b): Sappiamo che vale la seguente relazione

$$V \ldots \int^{b} E \, dl$$

$$V_A - V_B = \int_a^b E \, dl$$

$$\Rightarrow \quad V(R_2) - \underset{\overset{\parallel}{0}}{V_\infty} = \int_{R_2}^{\infty} E \, dl$$

$$\Rightarrow \quad V(R_2) = \int_{R_2}^{\infty} E \, dl$$

Quale E scegliamo?

Prendiamo la E data da $r > R_2$

cioè $\quad E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

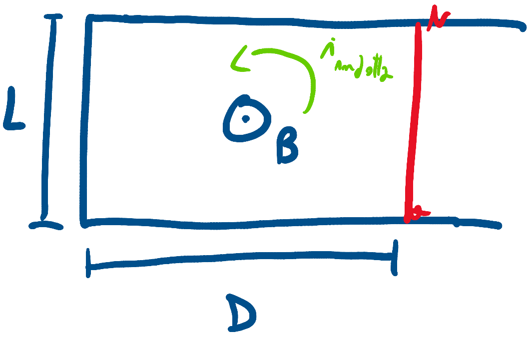
$$\Rightarrow \quad V(R_2) = \int_{R_2}^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2} \, dl = \int_{R_2}^{\infty} \frac{Q}{4\pi\varepsilon_0 r^2} \, dr$$

$dl = dr$ con abuso notazionale

$$= \frac{Q}{4\pi\varepsilon_0} \int_{R_2}^{\infty} \frac{1}{r^2} \, dr = \frac{Q}{4\pi\varepsilon_0} \left(-\frac{1}{r} \right) \Bigg|_{R_2}^{\infty}$$

$$= \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{R_2}$$

Es 3



$L = 10\ cm$

$D = 20\ cm$

$R = 100\ \Omega$

$B(t) = B_0 \left[1 - e^{-\alpha t}\right]$

$B_0 = 0{,}1\ T$

$\alpha = 0{,}1\ s^{-1}$

a) Determinare il senso di circolazione della corrente indotta e la sua intensità massima

b) Disegnare il grafico dell'intensità della corrente indotta in funzione del tempo

c) Determinare modulo, direzione e verso della forza che è necessario applicare sul lato mobile, affinché esso rimanga in quiete

Sol

a)

$$\xi = -\frac{d\Phi_S(B)}{dt}$$

$$\phi_S(B) = \int_S B\,\hat{n}\,ds = \int_S B\,ds = B \cdot S_{\text{spaziale}} =$$

la sbarra non si muove

$$= B \cdot L \cdot D$$

$$= B_0 \left[1 - e^{-\alpha t}\right] L\,D$$

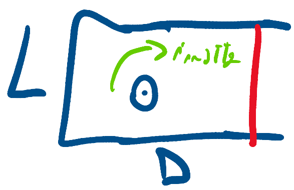
$$\Rightarrow \frac{\partial \phi_S(B)}{\partial t} = \frac{\partial}{\partial t}\left(B_0 L D - B_0 L D\, e^{-\alpha t}\right) =$$

$$= -B_0 L D\, e^{-\alpha t}(-\alpha) = B_0 L D \alpha\, e^{-\alpha t}$$

$$\xi = -\frac{d\phi_S(B)}{dt} = -B_0 L D \alpha\, e^{-\alpha t}$$

$$\Rightarrow i_{\text{indotta}} = \frac{\xi}{R} = -\frac{B_0 L D \alpha\, e^{-\alpha t}}{R}$$

gira in senso orario



$$|i_{\text{indotta}}| = \frac{B_0 L D \alpha\, e^{-\alpha t}}{R} \xrightarrow{t \to +\infty} 0$$

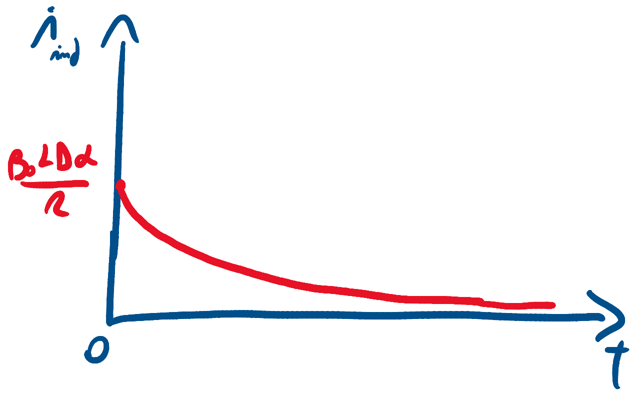
$$\frac{\partial}{\partial t} i_{\text{ind}} = -\frac{B_0 L D \alpha^2 e^{-\alpha t}}{R} < 0$$

max in $t = 0$

$$|i_{\text{ind}_{MAX}}| = \frac{B_0 L D \alpha}{R}$$

R

b):



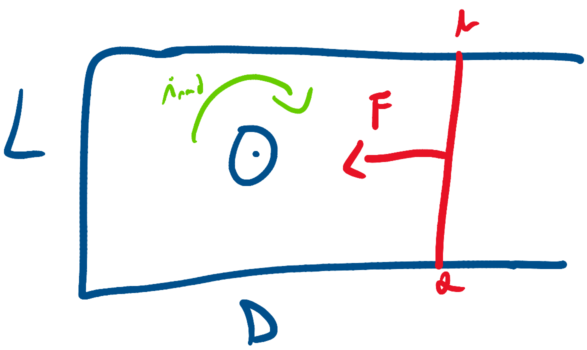
c): Uso la II Legge di Laplace

$$dF = i \, dl \wedge B$$

$$\Rightarrow F = \int_\gamma i \, B \, dl = i \, B \, L = i_{ind} \, L \, B =$$

lunghezza della sbarra

$$= -\frac{B_0 L D \alpha \, e^{-\alpha t}}{R} \cdot L \cdot B_0 (1 - e^{-\alpha t})$$



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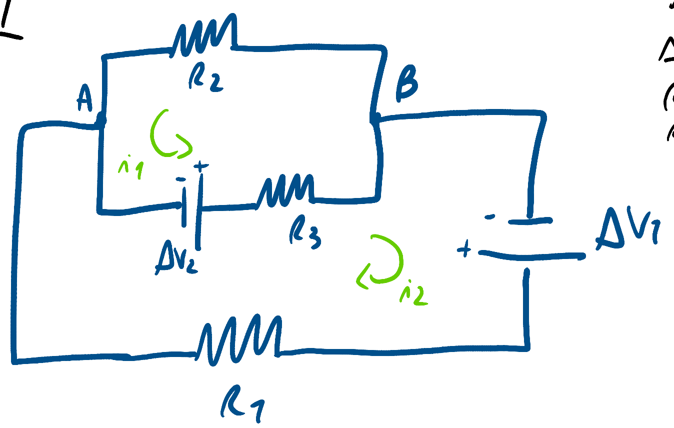
Appello 16/11/2021

sabato 13 gennaio 2024 15:17

Es 1



$\Delta V_1 = 9V$
$\Delta V_2 = 6V$
$R_1 = 1\Omega$
$R_2 = 3\Omega$
$R_3 = 1\Omega$

Trovare senso di circolazione e intensità delle correnti in ciascuna delle Resistenze

Sol

$$\begin{cases} -R_2 i_1 + \Delta V_2 - R_3 (i_1 - i_2) = 0 \\ \Delta V_1 - R_1 i_2 + \Delta V_2 - R_3 (i_2 - i_1) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -R_2 i_1 + \Delta V_2 - R_3 i_1 + R_3 i_2 = 0 \\ \Delta V_1 - R_1 i_2 + \Delta V_2 - R_3 i_2 + R_3 i_1 = 0 \end{cases}$$

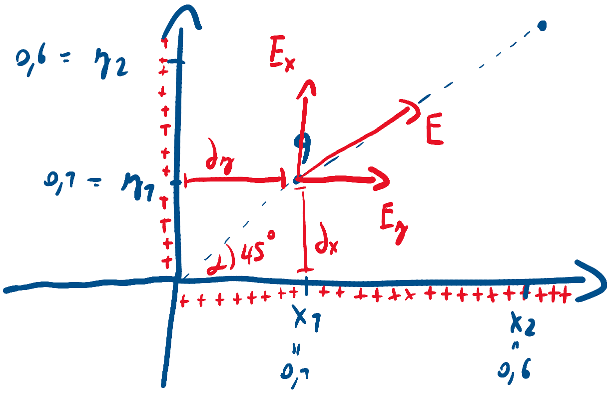
$$\Rightarrow \begin{cases} i_1(-R_2 - R_3) + \Delta V_2 + R_3 i_2 = 0 \\ \Delta V_1 - R_1 i_2 + \Delta V_2 - R_3 i_2 + R_3 i_1 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -4 i_1 + 6 + i_2 = 0 \\ 9 - i_2 + 6 - i_2 + i_1 = 0 \end{cases} \Rightarrow \begin{cases} 4 i_1 = 6 + i_2 \\ 15 - 2 i_2 + i_1 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} i_1 = \frac{6 + i_2}{4} \\ 15 - 2 i_2 + \frac{6 + i_2}{4} = 0 \end{cases} \Rightarrow \begin{cases} i_1 = \frac{6 + i_2}{4} \\ \frac{60 - 8 i_2 + 6 + i_2}{4} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} i_1 = \frac{6 + i_2}{4} \\ 66 - 7 i_2 = 0 \end{cases} \Rightarrow \begin{cases} i_1 = \frac{27}{7} \\ i_2 = \frac{66}{7} \end{cases}$$

Es 2



$\lambda = 8\ \frac{nC}{m}$

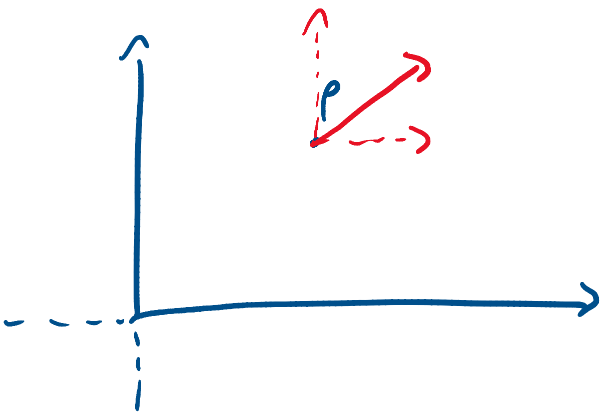
$q = 2\ \mu C$

$m = 1{,}2\ g$

Calcolare il Campo Elettrico nel punto $P_1 = (x_1, y_1)$

Calcolare la velocità del punto materiale, quando esso si è spostato, per effetto della Forza Elettrica, nel punto $P_2 = (x_2, y_2)$

Sol

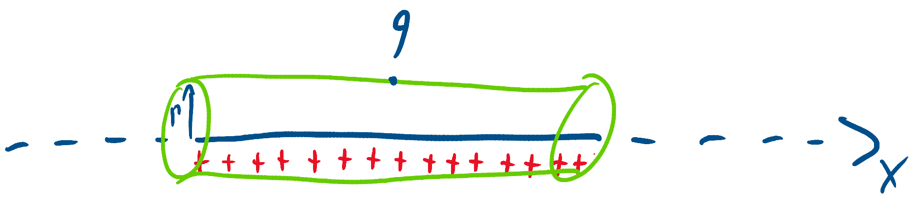


$$E_{TOT} = \sqrt{E_x^2 + E_y^2}$$

Dobbiamo trovare i campi elettrici di due fili infinitamente carichi

Usiamo il teorema di Gauss

$$\lambda = \frac{dQ}{dl}$$



$$\phi_S(E) = \int_S E\, ds = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$\underbrace{E \cdot S}_{\text{Superficie laterale del cilindro}} = \frac{Q}{\varepsilon_0}$$

$\Rightarrow$

$$E \cdot 2\pi r h = \frac{Q}{\varepsilon_0}$$

$L = h$ perché l'altezza del cilindro è proprio la lunghezza

$$\Rightarrow E = \frac{Q}{2\pi r h \varepsilon_0} = \frac{\lambda L}{2\pi r h \varepsilon_0} \overset{\downarrow}{=} \frac{\lambda h}{2\pi r h \varepsilon_0} = \frac{\lambda}{2\pi \varepsilon_0 r}$$

Dunque

$$E_x = \frac{\lambda}{2\pi \varepsilon_0 r} \quad {}^{\uparrow} r = 0{,}1$$

Analogamente

$$E_y = \frac{\lambda}{2\pi \varepsilon_0 r} \quad {}^{\uparrow} r = 0{,}1$$

$$\Rightarrow E_{tot} = \sqrt{E_x^2 + E_y^2} = \sqrt{2}\,\frac{\lambda}{2\pi \varepsilon_0 r} \quad {}^{\uparrow} r = 0{,}1$$

2): Sfruttiamo il fatto che

$$L_{TOT} = \Delta E_c$$

$\Rightarrow$

$$\frac{1}{2} m V_f^2 - \frac{1}{2} m V_i = \overset{\text{lavoro}}{L}$$

inoltre

$$L = \int_\gamma F \cdot dl \qquad e \qquad F = q E_0$$

Dunque

$$\frac{1}{2} m V_f^2 - \underset{0}{\frac{1}{2} m V_i^2} = \int_\gamma F\, dl = \int_\gamma q E\, dl = q \int_\gamma E\, dl$$

In questo caso γ è il tragitto da $P_1 = (x_1, y_1)$ fino a $P_2 = (x_2, y_2)$

$\Rightarrow$

$$\frac{1}{2} m V_f^2 = q \int_{x_1}^{x_2} E_x\, dx + q \int_{y_1}^{y_2} E_y\, dx$$

$$= q \int_{x_1}^{x_2} \frac{\lambda}{2\pi\varepsilon_0 r}\, dx + q \int_{y_1}^{y_2} \frac{\lambda}{2\pi\varepsilon_0 r}\, dy$$

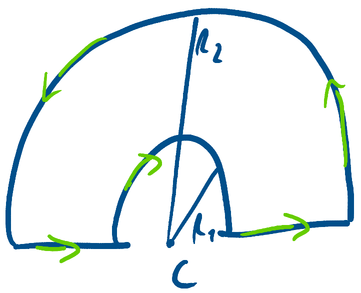
$r = x$

$$= q \int_{x_1}^{x_2} \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{x}\, dx + q \int_{y_1}^{y_2} \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{y}\, dy$$

$r = y$

$\Rightarrow$ Ricavo V_f e ho finito

Es 3



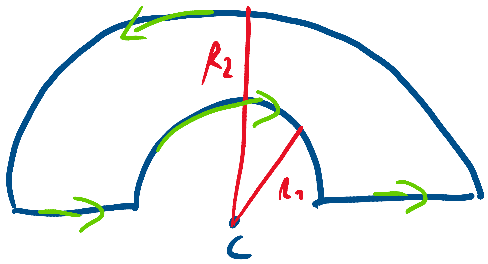
$i = 0{,}287\ A$

$R_1 = 3{,}15\ cm$

$R_2 = 7{,}8\ cm$

Calcolare il campo magnetico nel punto C

<u>SOL</u>



Dalla I Legge di Laplace ho che

$$dB = \frac{\mu_0 i}{4\pi}\,\frac{dl \wedge \vec{r}}{r^3}$$

inoltre i tratti orizzontali non danno contributo

Invece le due semicirconferenze SI :

La semicirconferenza di raggio R_2 produce un B diretto verso l'alto

$\odot\, B$

l'altra semicirconferenza di raggio R_1 lo produce verso il basso

$\otimes\, B$

Allora calcoliamo i due campi magnetici

quello di raggio R_1

Allora calcoliamo i due campi magnetici

Quello di raggio R_1

$$dB_1 = \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \wedge \vec{r}}{r^3}$$

$$\Rightarrow B_1 = \int_\gamma \frac{\mu_0 i}{4\pi} \frac{1}{R_1^2} dl = \int_\gamma \frac{\mu_0 i}{4\pi R_1^2} R_1 \, d\alpha =$$

$$= \frac{\mu_0 i}{4\pi R_1} \int_\gamma d\alpha = \frac{\mu_0 i}{4\pi R_1} \int_0^\pi d\alpha =$$

$$= \frac{\mu_0 i}{4\pi R_1} \cdot \pi = \frac{\mu_0 i}{4 R_1}$$

Calcoliamo B_2

$$dB_2 = \frac{\mu_0 i}{4\pi} \cdot \frac{d\vec{l} \wedge \vec{r}}{r^3}$$

$$\Rightarrow B_2 = \int_\gamma \frac{\mu_0 i}{4\pi} \frac{1}{R_2^2} dl = \int_\gamma \frac{\mu_0 i}{4\pi R_2^2} R_2 \, d\alpha = \int_0^\pi \frac{\mu_0 i}{4\pi R_2} d\alpha$$

$$= \frac{\mu_0 i}{4 R_2}$$

Dunque

$$B_{TOT} = B_1 - B_2$$

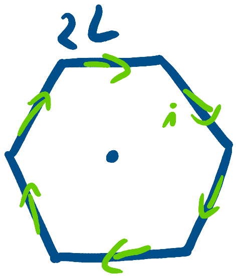
e sarà entrante perché $B_1 > B_2$

$\blacksquare$

Appello 18/02/2022

sabato 13 gennaio 2024 17:26

Es 1

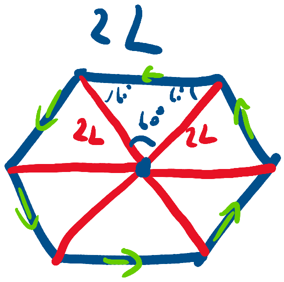


$L = 10\,cm$

$i = 12\,A$

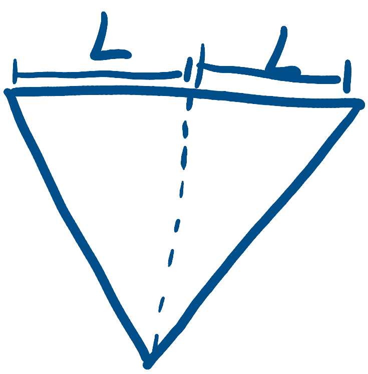
Calcolare il campo magnetico generato al centro della spira

Sol



$$dB = \frac{\mu_0 i}{4\pi} \frac{dl \wedge \bar{r}}{r^3}$$

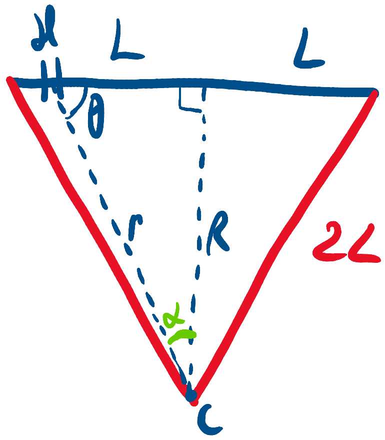
$$dB_1 = \frac{\mu_0 i}{4\pi} \frac{dl \wedge \bar{r}}{r^3}$$



$\Rightarrow$

$$B_1 = \int \frac{\mu_0 i}{4\pi} \frac{dl \cdot r \cdot \sin(\theta)}{r^3}$$

Ragioniamo per capire come scrivere meglio dl



$\theta = \frac{\pi}{2} - \alpha$

$\Rightarrow$

$\sin(\theta) = \sin\left(\frac{\pi}{2} - \alpha\right) =$

$= \cos(\alpha)$

Sappiamo che

$$\frac{L}{R} = tg(\alpha) \quad \Rightarrow \quad L = R\, tg(\alpha)$$

$$\Rightarrow \quad dl = R \frac{1}{\cos^2(\alpha)} d\alpha$$

Abbiamo trovato dl però c'è R che non sappiamo quanto vale

$\Rightarrow$ Dobbiamo riscrivere R in maniera più comoda

Si ha che per il Teorema di Pitagora

$$R = \sqrt{(2L)^2 - L^2} = \sqrt{4L^2 - L^2} = \sqrt{3}\, L$$

ma anche

$$R = r \cos(\alpha)$$

$$\Rightarrow \quad r = \frac{R}{\cos(\alpha)}$$

Allora

Allora

$$B_1 = \int \frac{\mu_0 i}{4\pi} \frac{dl \cdot r \cdot \sin(\theta)}{r^3} = \int \frac{\mu_0 i}{4\pi} \frac{\sin(\theta)}{r^2} dl =$$

$$= \int \frac{\mu_0 i}{4\pi} \cos(\alpha) \cdot \frac{1}{r^2} dl = \int \frac{\mu_0 i}{4\pi} \cos(\alpha) \cdot \frac{1}{r^2} \cdot R \frac{1}{\cos^2(\alpha)} d\alpha$$

$$= \int \frac{\mu_0 i}{4\pi} \cdot \frac{1}{r^2} \cdot R \cdot \frac{1}{\cos(\alpha)} d\alpha =$$

$$= \int \frac{\mu_0 i}{4\pi} \cdot \frac{\cos^2(\alpha)}{R^2} \cdot R \frac{1}{\cos(\alpha)} d\alpha =$$

$$= \int \frac{\mu_0 i}{4\pi} \cdot \frac{\cos(\alpha)}{R} d\alpha = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\mu_0 i}{4\pi} \cdot \cos(\alpha) \cdot \frac{1}{R} d\alpha$$

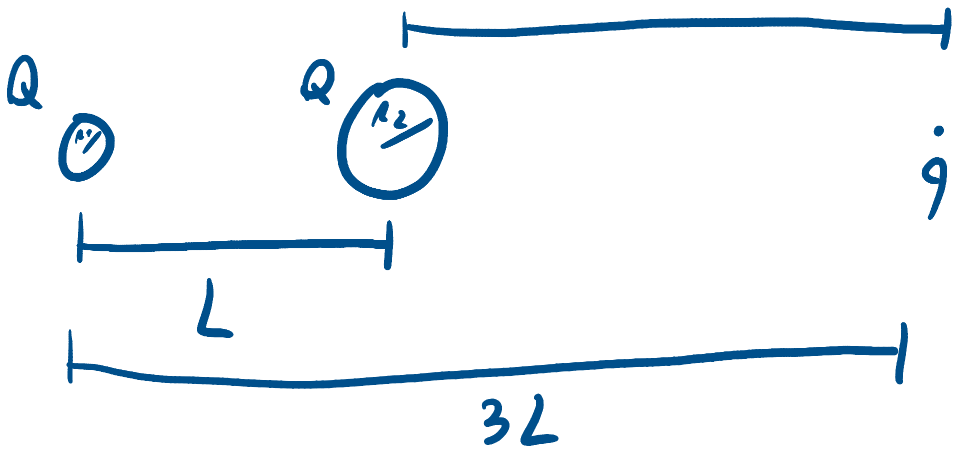
"$60°/2$"

$$= \frac{\mu_0 i}{4\pi R} \cdot \sin(\alpha) \Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$$

Dunque i lati sono 6

$$B_{TOT} = 6 B_1$$

Es 2



$R_1 = 1cm$

$R_2 = 3cm$

$L = 2m$

$Q = 2 \cdot 10^{-3} C$

$q = -2 \cdot 10^{-6} C$

a) Calcolare la forza esercitata su q

b) La carica q viene portata all'infinito. Calcolare il lavoro compiuto dalle forze elettrostatiche

Sol

a):

$$F_{TOT} = F_1 + F_2$$

$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q \cdot q}{(3L)^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q \cdot q}{(2L)^2}$$

b)

$$L = -\Delta U$$

$$- \frac{Q q_0}{\ldots} \left(\frac{1}{\ldots} - \frac{1}{\ldots} \right) =$$

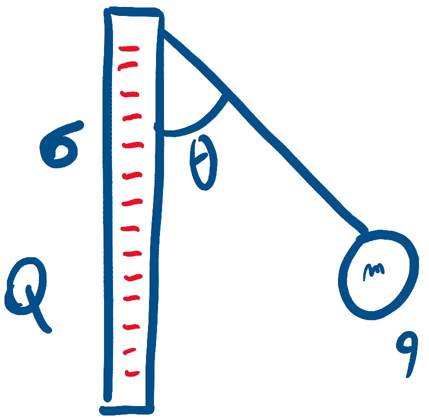
$$\Delta U = U_f - U_i = \frac{Q q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right) =$$

$$= \frac{Q q_0}{4\pi\varepsilon_0} \underbrace{\frac{1}{r_f}}_{\substack{\lim_{r_f \to +\infty} \\ 0}} - \frac{Q q_0}{4\pi\varepsilon_0} \cdot \frac{1}{r_i}$$

$\Rightarrow$

$$L = \frac{Q q_0}{4\pi\varepsilon_0} \frac{1}{r_i} = \frac{Q q_0}{4\pi\varepsilon_0} \cdot \frac{1}{3L}$$

Es 3



$m = 1{,}12 \ mg$

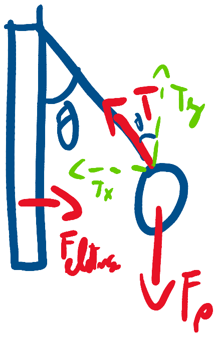
$q = 19{,}7 \ nC$

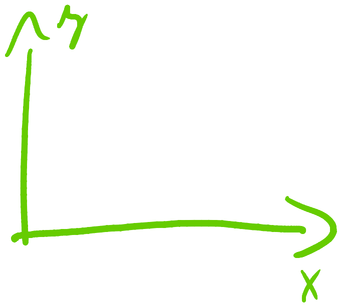
$\theta = 30°$

Determinare σ

Sol

$$\sigma = \frac{\partial Q}{\partial S}$$





All'equilibrio $F_{TOT} = 0$

$$\Rightarrow \begin{cases} F_{TOT_x} = 0 \\ F_{TOT_y} = \end{cases}$$

$$\Rightarrow \begin{cases} F_{elettrica} - T_x = 0 \\ -F_p + T_y = 0 \end{cases} \Rightarrow \begin{cases} F_{elettrica} = T_x \\ T_y = mg \end{cases} \qquad \begin{pmatrix} T_x = T\sin(\theta) \\ T_y = T\cos(\theta) \end{pmatrix}$$

$$\Rightarrow \begin{cases} F_{elettr} = T\sin(\theta) \\ T\cos(\theta) = mg \end{cases}$$

Formula del Campo Elettrico per un piano

$$F_{elett} = E \cdot q = \boxed{\frac{\sigma}{2\varepsilon_0}} \cdot q$$

Quindi

F_{elett} $\sigma \cdot q$

$$F_{elett} = \frac{\sigma}{\varepsilon_0} \cdot q$$

Sappiamo che

$$\frac{T_x}{T_y} = tg(\theta)$$

$$\Rightarrow \frac{F_{elett}}{F_p} = tg(\theta)$$

$$\Rightarrow \frac{F_{elett}}{m \cdot g} = tg(\theta)$$

$$\Rightarrow \frac{\sigma}{2\varepsilon_0} \cdot q = m \cdot g \cdot tg(\theta)$$

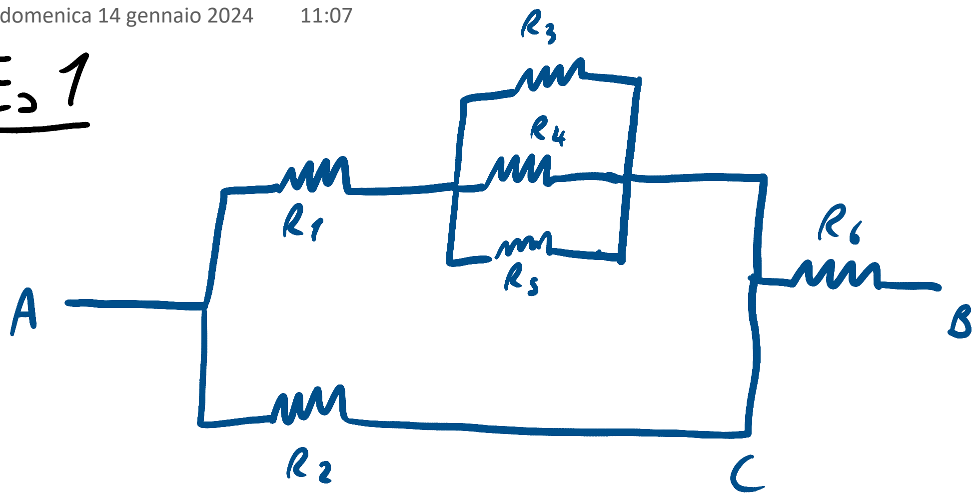
$$\Rightarrow \sigma = \frac{m \cdot g \cdot tg(\theta) \cdot 2\varepsilon_0}{q}$$

$\blacksquare$

Appello 15/03/2022

domenica 14 gennaio 2024 11:07

Es 1



$R_1 = 3$

$R_2 = 20$

$R_3 = 12$

$R_4 = 6$

$R_5 = 4$

$R_6 = 5$

$\Delta V_{AB} = 5{,}4$

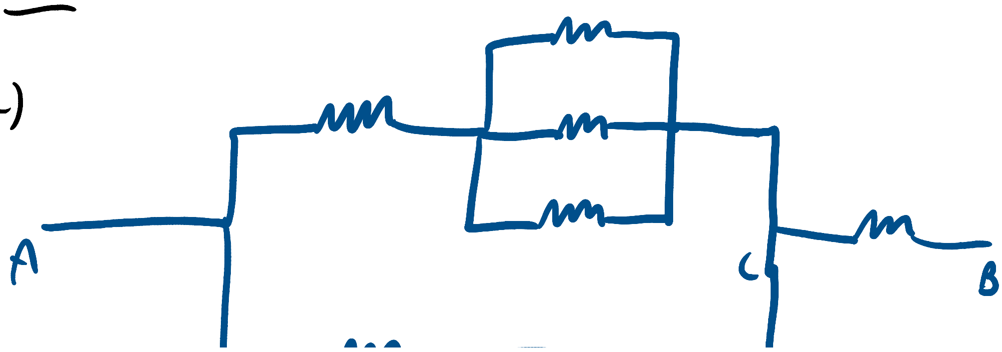
a) Calcolare il valore della Resistenza vista ai capi del circuito (A, B)

b) Calcolare la corrente che circola nella resistenza R_6

c) Calcolare la potenza dissipata dalla resistenza R_6

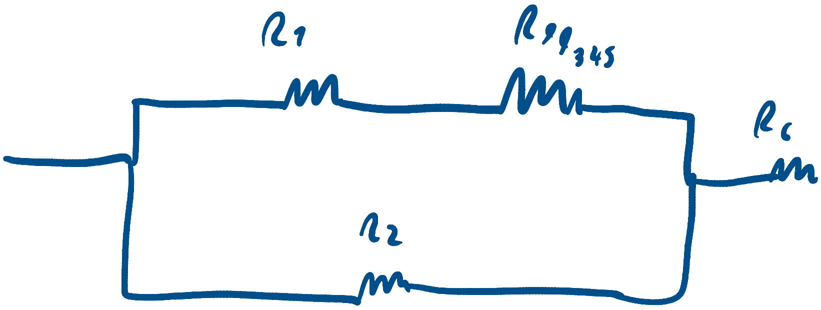
Sol

a)

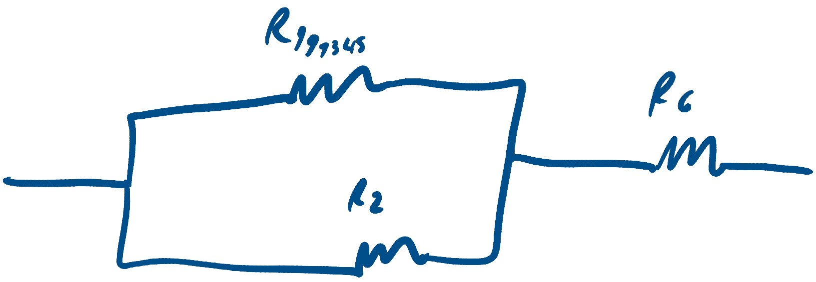


$$\frac{1}{R_{eq_{345}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

$$\Rightarrow R_{eq_{345}} = \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)^{-1}$$



$$R_{eq_{1345}} = R_1 + R_{eq_{345}}$$



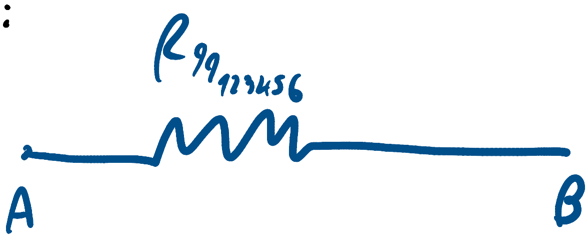
$$\frac{1}{R_{eq_{12345}}} = \frac{1}{R_{eq_{1345}}} + \frac{1}{R_2}$$

$$\Rightarrow R_{eq_{12345}} = \left(\frac{1}{R_{eq_{1345}}} + \frac{1}{R_2}\right)^{-1}$$

$$R_{eq_{123456}} = R_{eq_{12345}} + R_6$$

$$R_{eq_{123456}} = R_{eq_{12345}} + R_6$$

b):



$i_6 = i_{12345}$ (le Resistenze sono in Serie) $= i_{TOT}$

Sappiamo che vale

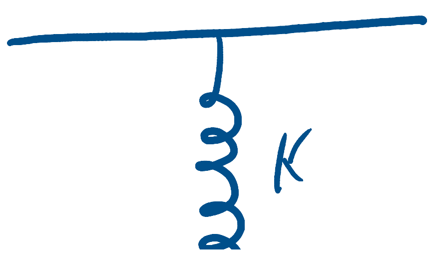
$$\Delta V = R \cdot i$$

$$\Rightarrow i_{TOT} = \frac{\Delta V_{AB}}{R_{eq_{123456}}} \quad \Rightarrow \quad i_6 = i_{TOT}$$

c)

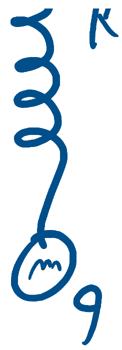
$$W = i_6^2 \cdot R_6 = i_{TOT}^2 \cdot R_6$$

Es 3



$m = 0{,}1\,kg$

$q = 4\pi \cdot 10^{-9}\,C$

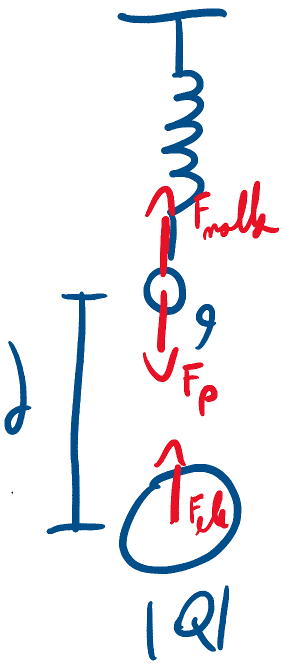


$q = 4\pi \cdot 10$ C

K = costante elastica

Calcolare segno e distanza da q della carica di valore assoluto $|Q| = 8{,}85 \cdot 10^{-5}$ C affinché l'allungamento della molla sia nullo

SOL



Q e q hanno stesso segno

$$F_{TOT} = F_{molla} + F_{ele} - F_p$$

alla fine c'è equilibrio con allungamento della molla nullo

Alla fine c'è equilibrio con allungamento della molla nullo

$$F_{TOT} = 0$$

$$\Rightarrow$$

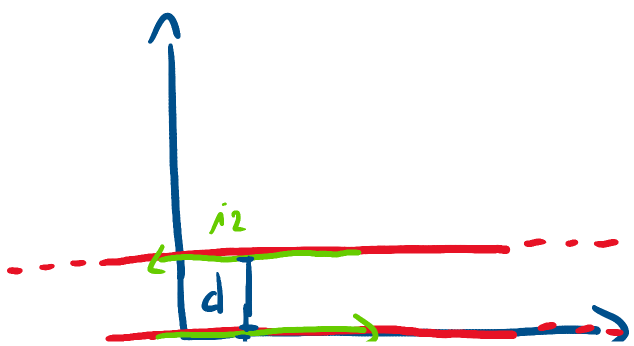
$$\underbrace{F_{molla}}_{0} + F_{ele} = F_p$$

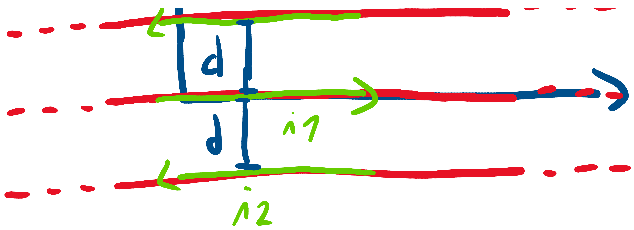
$$\Rightarrow \quad F_{ele} = F_p$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = mg$$

$$\Rightarrow \quad r = \sqrt{\frac{Qq}{4\pi\varepsilon_0}\frac{1}{mg}}$$

Es 2





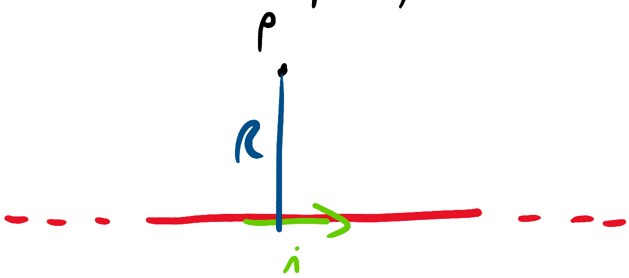
Ricavare l'espressione per il campo magnetico prodotto da ogni filo ad una generica distanza r da esso e determinare:

a) Il campo magnetico generato dai conduttori in $P_1 = (0, 2d, 0)$

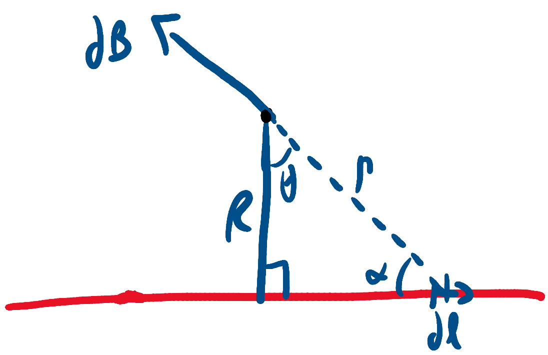
b) La forza per unità di lunghezza sul conduttore centrale

Sol

Consideriamo un filo qualsiasi



Allora



$R = r \cdot \cos(\theta) \Rightarrow r = \frac{R}{\cos(\theta)}$

$tg(\theta) = \frac{l}{R} \Rightarrow l = R\, tg(\theta)$

$\Rightarrow dl = R \frac{1}{\cos^2(\theta)} d(\theta)$

$$dB = \frac{\mu_0 i}{4\pi} \frac{dl \wedge r}{r^3} = \frac{\mu_0 i}{4\pi} \frac{dl \cdot r \cdot \sin(\alpha)}{r^3}$$

$$dB = \frac{\mu_0 i}{4\pi} \frac{dx \times r}{r^3} = \frac{\mu_0 i}{4\pi} \frac{dx \cdot r \sin(\alpha)}{r^3}$$

$$= \frac{\mu_0 i}{4\pi} \cdot \frac{1}{r^2} \cos(\theta)\, dl$$

$\alpha = \frac{\pi}{2} - \theta$

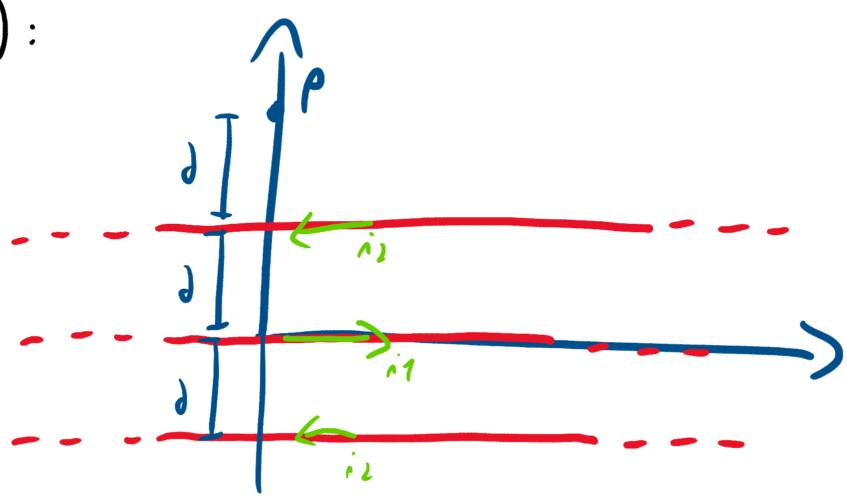
$\Downarrow$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$

$$B = \frac{\mu_0 i}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(\theta)}{R^2} \cdot \cos(\theta) \cdot R \cdot \frac{1}{\cos^2(\theta)}\, d\theta =$$

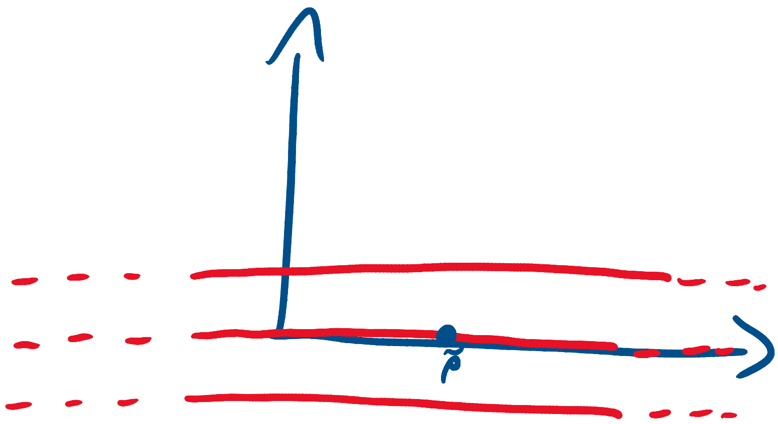
$$= \frac{\mu_0 i}{4\pi R} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\theta)\, d\theta = \frac{\mu_0 i}{2\pi R}$$

a) :



$$B_{TOT} = -\frac{\mu_0 i_2}{2\pi d} + \frac{\mu_0 i_1}{2\pi\, 2d} - \frac{\mu_0 i_2}{2\pi\, 3d}$$

b)



$$F_{TOT} = F_{\substack{filo\\superiore}} + \underbrace{F_{\substack{filo\\centrale}}}_{=0} + F_{\substack{filo\\inferiore}} = F_{\substack{filo\\superiore}} + F_{\substack{filo\\inferiore}} = 0$$

perché la distanza tra filo e punto è zero (il punto sta sul filo)

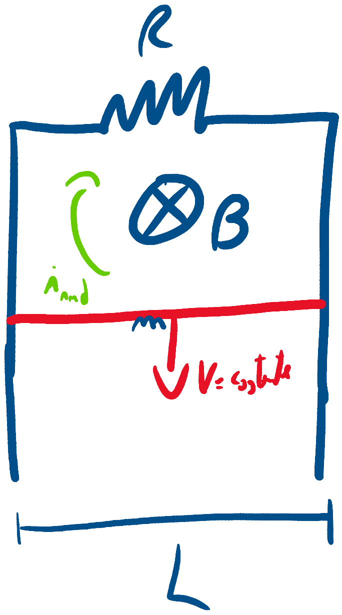
per simmetria

∎

Appello 7/09/2022

domenica 14 gennaio 2024 22:39

Es 1



$L = 1\,m$

$m = 50\,g$

$R = 5\,\Omega$

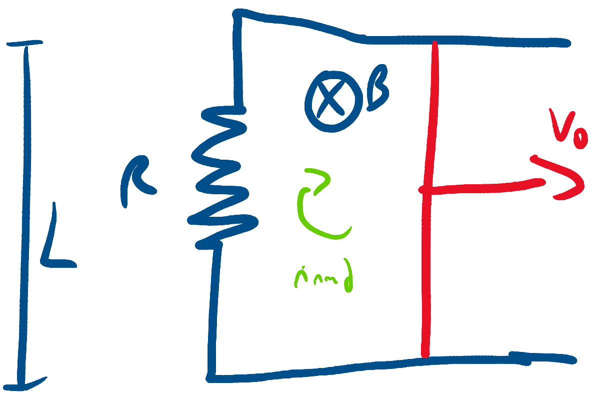
$B = 0{,}5\,T$

Calcolare

a) Verso e intensità della corrente indotta

b) il modulo V_0 della velocità con cui la barretta cade

SOL



$$\xi = -\frac{d\Phi_S(B)}{dt}$$

$x(t) = S_0 + V \cdot t$

$$\phi_s(B) = \int_S B \, ds = B \cdot S_{superficie} = B \, (S_0 + V \cdot t) \cdot L$$

$$\frac{\phi_s(B)}{dt} = B L V$$

$$\xi = - B L V$$

$$i_{ind} = \frac{\xi}{R} = - \frac{B L V}{R}$$

$\Rightarrow$ gira in senso antiorario

b)

$$F_{TOT} = 0$$

$$\Rightarrow \quad F_{\substack{CAMPO \\ MAGNETICO}} = F_p$$

II Legge di Laplace:

$$dF = i \, dl \wedge B$$

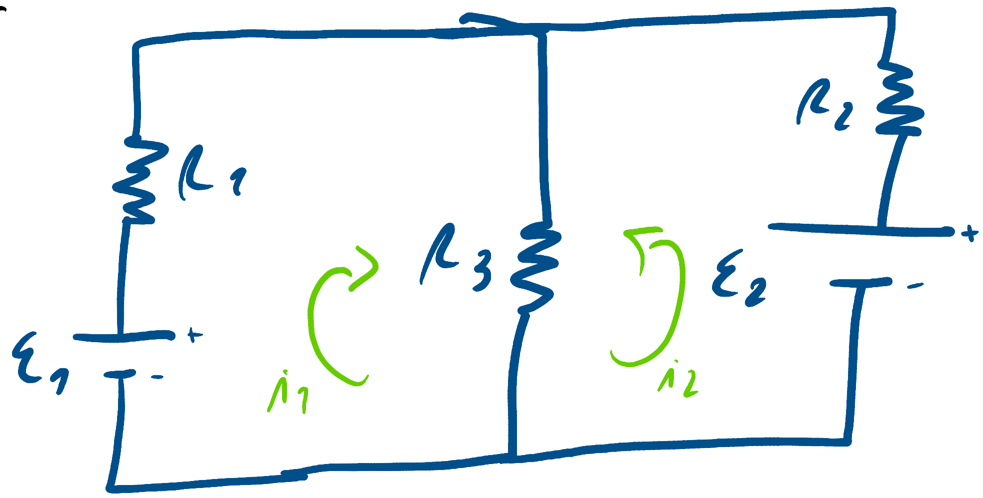
$$\Rightarrow F = i\,BL$$

Donc que

$$\frac{BLV}{R} \cdot BL = mg$$

$$\Rightarrow \frac{B^2L^2V}{R} = mg \Rightarrow V = \frac{mgR}{B^2L^2}$$

Ex 2



$\mathcal{E}_1 = 11V$

$\mathcal{E}_2 = 7V$

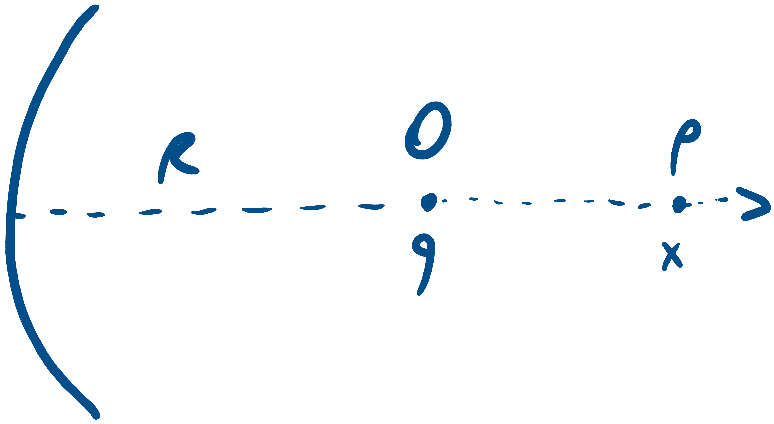
$R_1 = 2\Omega$

$R_2 = 1\Omega$

$R_3 = 1\Omega$

$$\begin{cases} \mathcal{E}_1 - R_1 i_1 - R_3(i_1 + i_2) = 0 \\ \mathcal{E}_2 - R_2 i_2 - R_3(i_2 + i_1) = 0 \end{cases}$$

Es 3



$q = 7\,nC$

$R = 7\,m$

λ

$a = 72,7\ m/s^2$

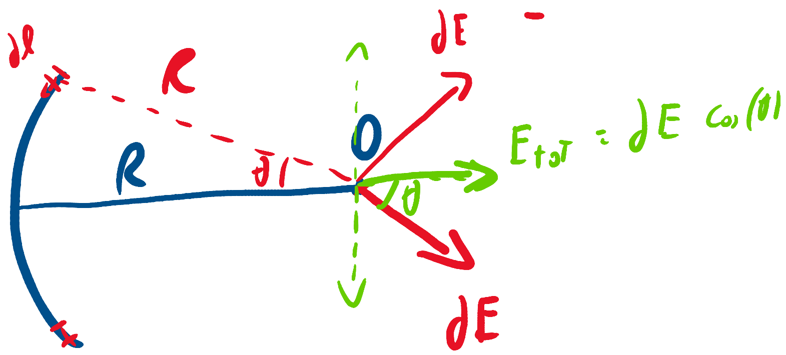
$V_P = 4,47\ m/s$

$\Delta V_{OP} = 70\,kV$

a) Ricavare l'espressione del modulo del campo elettrico in O e direzione e verso

b) Determinare valore di λ e della massa m di q

Sol



$$L = \frac{1}{4} \cdot 2\pi R = \frac{\pi}{2}$$

$\bar{2}$

$$dE_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\cos(\theta)$$

$\lambda = \frac{dQ}{dl} \Rightarrow dQ = \lambda\, dl$

$\Rightarrow$

$$dE_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{r^2}\cos(\theta)$$

$\Rightarrow$

$$E_{tot} = \frac{1}{4\pi\varepsilon_0}\cdot\lambda\int\frac{1}{R^2}\cos(\theta)dl = \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{R^2}\int R\cos(\theta)\,d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{R}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos(\theta)\,d\theta =$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}\left(\sin(\theta)\right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\lambda}{4\pi\varepsilon_0 R}\cdot\sqrt{2}$$

b)

$$\Delta E_{c_{tot}} = L_{lavoro}$$

$$\frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 = L_{\text{avoro}}$$

$$\frac{1}{2} m V_f = \int_\gamma F\, dl \underset{F = E \cdot q}{=} \int_\gamma E \cdot q\, dl = q \int_\gamma E\, dl$$

$$= q \int_0^P E\, dl = q \cdot \Delta V_{op}$$

$$\Rightarrow \quad m = \frac{q \cdot \Delta V_{op} \cdot 2}{\underset{= V_p}{V_f^2}}$$

Inoltre

$$F = m \cdot a$$

e

$$F = E \cdot q = \frac{\sqrt{2}\, \lambda}{4 \pi \varepsilon_0 R} \cdot q$$

$$\Rightarrow \quad q \frac{\sqrt{2}\, \lambda}{4 \pi \varepsilon_0 R} = m \cdot a$$

$$\Rightarrow \quad \lambda = \frac{m \cdot a}{\sqrt{2}} \cdot \frac{4 \pi \varepsilon_0 R}{q}$$

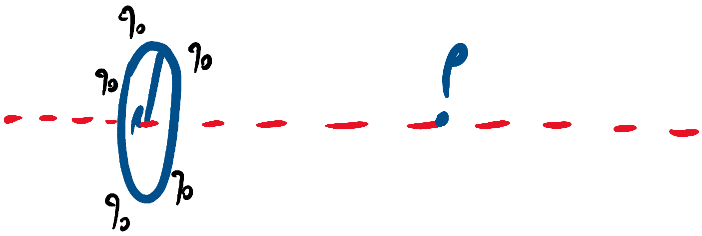
V2 9

Appello 30/06/2022

lunedì 15 gennaio 2024 11:03

Es 1

$R = 6\,cm$



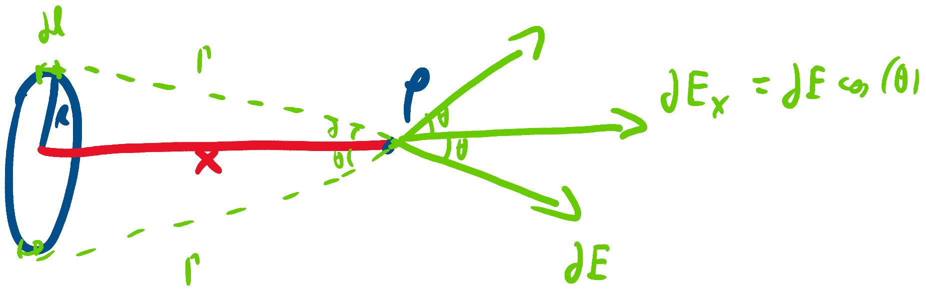
Ricavare l'espressione del campo elettrico generato dall'anello e in seguito determinare:

a) la distanza x di P dal centro dell'anello

b) il potenziale in P

c) l'energia potenziale di una carica $q = \frac{q_0}{100}$ posta in P

Sol



$$dE_x = dE \cos(\theta)$$

$$\Rightarrow dE_x = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r^2} \cos(\theta)$$

Poiché

$$dQ = \lambda\, dl$$

Poiché

$$dQ = \lambda \, dl$$

$\Rightarrow$

$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{\lambda \, dl}{r^2} \cos(\theta)$$

Inoltre

$$\lambda = \frac{Q}{l} \Rightarrow \lambda = \frac{q_0}{2\pi R}$$

$\Rightarrow$

$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \cdot \frac{q_0}{2\pi R} \cdot \cos(\theta) \, dl$$

ma osservando il triangolo che si viene a creare notiamo che

$$r = \sqrt{x^2 + R^2}$$

e sempre per lo stesso triangolo risulta che

$$x = r\cos(\theta) \Rightarrow \cos(\theta) = \frac{x}{r} = \frac{x}{\sqrt{x^2+R^2}}$$

Dunque

$$dE_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{(x^2+R^2)} \cdot \frac{q_0}{2\pi R} \cdot \frac{x}{\sqrt{x^2+R^2}} \, dl$$

$\Rightarrow$

$$E_x = \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{(x^2+R^2)^{3/2}} \cdot \frac{q_0}{2\pi R} \cdot x \, dl$$

$$E_x = \int_{\gamma_{\text{Anello}}} \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{(x^2+R^2)^{3/2}} \cdot \frac{q_0}{2\pi R} \cdot x \, dl$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{(x^2+R^2)^{3/2}} \cdot \frac{q_0}{2\pi R} \cdot x \cdot 2\pi R$$

$$= \frac{q_0 \, x}{4\pi\varepsilon_0 \, (x^2+R^2)^{3/2}}$$

a):

$$\frac{\partial}{\partial x} E_x = \frac{q_0}{4\pi\varepsilon_0} \left(\frac{(x^2+R^2)^{3/2} - x \, \frac{3}{\cancel{2}} (x^2+R^2)^{1/2} \cdot \cancel{2} x}{(x^2+R^2)^3} \right)$$

$$= \frac{q_0}{4\pi\varepsilon_0} \left(\frac{(x^2+R^2)^{3/2} - 3x^2 (x^2+R^2)^{1/2}}{(x^2+R^2)^3} \right)$$

Pongo

$$\frac{\partial}{\partial x} E_x = 0$$

$$\Rightarrow \frac{q_0}{4\pi\varepsilon_0} \left(\frac{(x^2+R^2)^{1/2} \left((x^2+R^2) - 3x^2 \right)}{(x^2+R^2)^3} \right) = 0$$

$$\Leftrightarrow \quad x^2 + R^2 - 3x^2 = 0 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad R^2 - 2x^2 = 0$$

$\Leftrightarrow \quad R^2 - 2x^2 = 0$

$\Rightarrow 2x^2 = R^2 \Rightarrow x^2 = \frac{R^2}{2}$

$\Rightarrow x = \frac{R}{\sqrt{2}}$

b)

$$V_{(P)} - \underset{0}{\underset{\shortparallel}{V_\infty}} = \int_P^{+\infty} E \, dl$$

$$\Rightarrow V_{(P)} = \int_x^{+\infty} E \, dl = \int_x^{+\infty} \frac{q_0 x}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}} \, dx$$

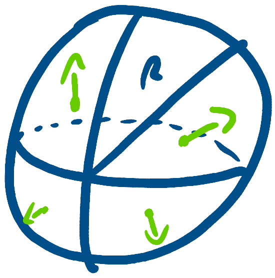
$$= \frac{q_0}{4\pi\varepsilon_0} \int_x^{+\infty} \frac{x}{(x^2+R^2)^{3/2}} \, dx =$$

$$= \frac{q_0}{4\pi\varepsilon_0} \left(- \frac{1}{(x^2+R^2)^{1/2}} \right) \Bigg|_x^{+\infty} =$$

$$= \frac{q_0}{4\pi\varepsilon_0} \frac{1}{(x^2+R^2)^{1/2}}$$

c): $U_{(P)} = V_{(P)} \cdot q$

Es 2



$R = 10\,cm$

$|E| = K r^2$ r distanza dal centro della sfera

Calcolare

a) segno e valore della carica posseduta dalla sfera

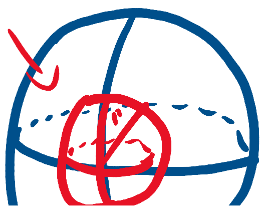
b) Potenziale elettrostatico al centro della sfera

c) la velocità con cui arriva al centro un elettrone che parte da fermo dalla superficie

Sol

a): Segno positivo

Superficie di Gauss $r < R$



Ho che

$$\oint_S E \hat{n}\, ds = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$E \cdot S_{superficie\ gauss} = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$K r^2 \cdot 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$4\pi K r^4 = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow Q_{int} = 4\pi K r^4 \varepsilon_0$$

Per avere la Q_{TOT} pongo $r = R$

$$\Rightarrow Q_{TOT} = 4\pi K R^4 \varepsilon_0$$

b)

$$V_{(0)} - V_{\infty} = \int_0^{+\infty} E\, dl = \int_0^R E\, dl + \int_R^{+\infty} E\, dl$$

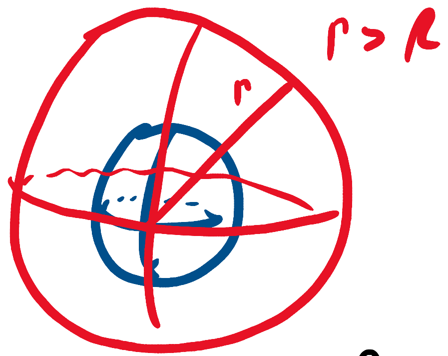
$$\Rightarrow \int^R \ldots dl + \int^{+\infty} E\, dl$$

$\Rightarrow$

$$V_{(0)} = \int_0^R E\,dl + \int_R^{+\infty} E\,dl$$

All'interno della sfera $E = K r^2$

Per trovare E al di fuori della sfera usiamo nuovamente Gauss



$$\phi_S(E) = \int_S E\,ds = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$ $$\int_S E\,ds = \frac{Q_{della\ sfera\ blu}}{\varepsilon_0} = \frac{Q_{TOT}}{\varepsilon_0}$$

$\Rightarrow$ $$E \cdot S_{sfera} = \frac{Q_{TOT}}{\varepsilon_0}$$

$\Rightarrow$ $$E \cdot 4\pi r^2 = \frac{Q_{TOT}}{\varepsilon_0} \quad \Rightarrow E = \frac{Q_{TOT}}{4\pi\varepsilon_0 r^2}$$

Dunque

$$V_{(0)} = \int_0^R K r^2\,dr + \int_R^{+\infty} \frac{Q_{TOT}}{4\pi\varepsilon_0 r^2}\,dr$$

$$= K \frac{r^3}{3}\Big|_0^R + \frac{Q_{TOT}}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_R^{+\infty} =$$

$$= K\frac{R^3}{3} + \frac{Q_{TOT}}{4\pi\varepsilon_0}\cdot\frac{1}{R}$$

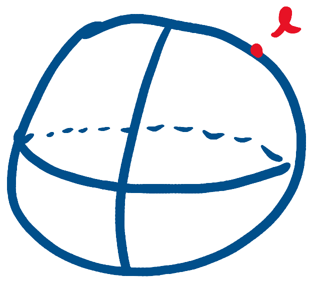
Ricordiamo che:

$$Q_{TOT} = 4\pi K R^4 \varepsilon_0$$

$\Rightarrow$

$$V(0) = K\frac{R^3}{3} + K R^3 = \frac{4}{3} K R^3$$

c):



carica elettrone

$$\Delta E_c = L = \int_\gamma F \cdot dl = \int_\gamma E \cdot e \, dl$$

$\Rightarrow$

$$\frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 = e \int_R^0 K r^2 \, dr$$

$\Rightarrow$

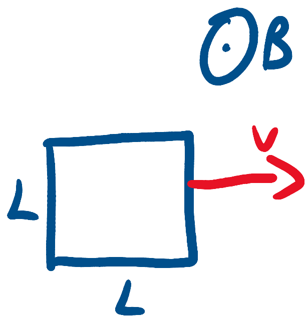
$$\frac{1}{2} m V_f^2 = e K \frac{r^3}{3}\Big|_R^0$$

elettrone ha carica negativa

$$\Rightarrow \quad \frac{1}{2} m V_f^2 = e K\left(-\frac{R^3}{3}\right) = -eK\frac{R^3}{3} = |e| K \frac{R^3}{3}$$

$$\Rightarrow \quad V_f = \sqrt{|e| K \frac{R^3}{3} \cdot 2 \cdot \frac{1}{m}}$$

Es 3



$B(x) = \alpha + \beta x$

$V = 0{,}1\ m/s$

$L = 10\ cm$

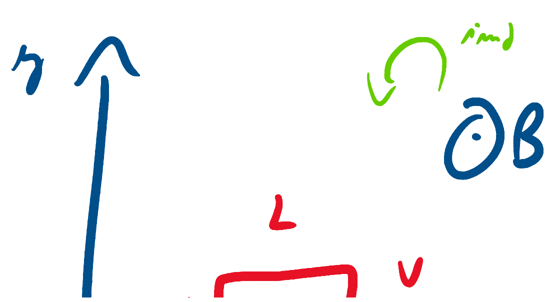
$\beta = 0{,}1\ T/m$

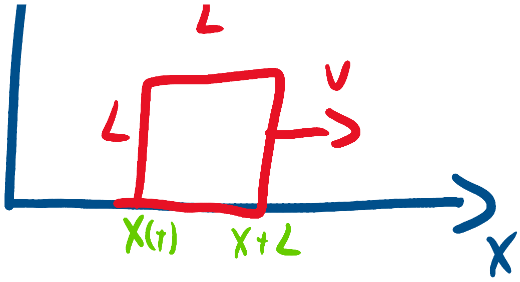
$R = 1\ \Omega$

Determinare il valore della corrente indotta

Quanto deve valere la forza con cui viene trascinata la spira per mantenere la velocità costante?

Sol





$$\phi_s(B) = \int_S B\, ds = \int_x^{x+L} B \cdot L\, dx = \int_x^{x+L} L(\alpha + \beta x)\, dx$$

$$= L\left(\alpha x + \beta \frac{x^2}{2}\right)\Bigg|_x^{x+L} =$$

$$= L\left(\alpha(x+L) - \alpha x + \frac{\beta}{2}(x+L)^2 - \frac{\beta}{2}x^2\right) =$$

$$= L\left(\cancel{\alpha x} + \alpha L - \cancel{\alpha x} + \frac{\beta}{2}(x^2 + L^2 + 2xL) - \frac{\beta}{2}x^2\right) =$$

$$= L\left(\alpha L + \cancel{\frac{\beta}{2}x^2} + \frac{\beta}{2}L^2 + \beta x L - \cancel{\frac{\beta}{2}x^2}\right) =$$

$$= L\left(\alpha L + \frac{\beta}{2}L^2 + \beta x L\right) =$$

$$= \alpha L^2 + \frac{\beta}{2}L^3 + \beta L^2 x$$

Questo x dipende dal tempo

$$\Rightarrow \phi_s(B) = \alpha L^2 + \frac{\beta}{2}L^3 + \beta L^2 x$$

$$\frac{d\phi_s(B)}{dt} = \beta L^2 v$$

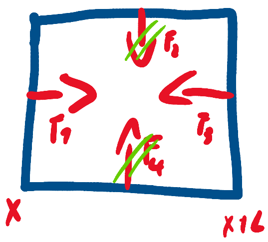
v(t)

$$\xi = -\frac{d\phi_s(B)}{dt} = -\beta L^2 v$$

$$\xi = -\frac{d\Phi_B(t)}{dt} = -\beta L v$$

$$i_{ind} = \frac{\xi}{R} = -\frac{\beta L^2 v}{R}$$

$$dF = i\, d\vec{l} \wedge \vec{B}$$



F_2 e F_4 per simmetria si annullano

$$F_1 = i B L$$

$$F_2 = i B L$$

$$\Rightarrow F_1 = i(\alpha + \beta x)L$$

$$F_3 = i(\alpha + \beta(x+L))L$$

$$\Rightarrow F_{TOT} = F_1 - F_3$$